【基础理论研究】微分方程与动力系统专题

Volterra 模糊积分方程的数值解

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摘　要:采用 Sumudu 分解法对 Volterra 模糊积分方程进行了数值求解,并通过详细的数值实例,验证了所采用方法的有效性与实用性。

关键词: Volterra 积分方程;Sumudu 分解法;模糊积分方程;数值解;Sumudu 变换

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0　引言

1982 年,Dubois 与 Prade 首次提出了模糊函数的理论框架[1],该理论一经提出即吸引了学术界的广泛关注并展开了深入研究[2-3]。随着时间的推移,模糊微积分方程在不确定性建模领域的动态系统分析中显示出其不可或缺的地位[4-5]。文献[6]应用同伦摄动 Sumudu 变换技术,成功求解了非线性模糊微积分方程的数值解,但该方法的应用前提是必须明确初始条件以及源函数的模糊参数设定。文献[7]将 Adomian 分解法引入非线性分数阶 Fredholm 积分方程的求解中,通过级数解的形式给出了方程的解,通过分析级数解的收敛性,验证了该方法的有效性与实用性。文献[8]结合 Adomian 多项式与积分方程技术,对非线性分数阶 Volterra 积分方程的求解进行了探讨,并通过数值示例展示了该方法的可行性与有效性,然而在计算较多 Adomian 多项式时,该方法可能引入特定的误差。进一步的研究成果与讨论见文献[9-11]。基于上述研究,本文采用 Sumudu 分解法对 Volterra 模糊积分方程进行了数值求解。首先,将问题通过参数化方式表述;然后,对 Volterra 模糊积分方程应用模糊 Sumudu 变换,以获得对应的等价方程;最后,通过数值实例,验证了 Sumudu 分解法的有效性和实用性。

1　预备知识

定义 1[12-13]　设 E 是上半连续的所有模糊数的紧集,则 Y 的 ρ 阶水平截集为

$$[Y]_{\rho}=\{t\in \mathbf{R}\,|\,Y(t)\geqslant\rho,0<\rho\leqslant1\}。$$

为了书写方便,将 $[Y]_{\rho}$ 记为 $Y(\rho)$,其左右端点分别记为 $\underline{Y}(\rho)$、$\bar{Y}(\rho)$ 则

$$Y(\rho)=[\underline{Y}(\rho),\bar{Y}(\rho)]。$$

考虑映射 $\bar{d}:L(R)\times L(R)\rightarrow\mathbf{R}$,

$$d(Y,V)=\sup_{0<\rho\leqslant1}\max\{|\underline{Y}(\rho)-\underline{V}(\rho)|,|\bar{Y}(\rho)-\bar{V}(\rho)|\},$$

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其中，$Y(\rho)=[\underline{Y}(\rho),\bar{Y}(\rho)]$，$V(\rho)=[\underline{V}(\rho),\bar{V}(\rho)]$。则 d 为 $L(R)$ 上的一个变量，且满足条件：

（ⅰ）$d(Y+W,V+W)=d(Y,V)$；（ⅱ）$d(kY,kV)=|k|d(Y,V)$；（ⅲ）$d(Y+W,W+e)\leqslant d(Y,W)+d(V,e)$。其中，$X,Y,W,e\in L(R)$，$R\in\mathbf{R}$。

定义 2[12]　令 $f:\mathbf{R}\to L(R)$，若对任意 $x_0\in R$ 存在 $f'(x_0)\in L(R)$，则

$$f'(x_0)=\lim_{h\to0^+}\frac{f(x_0+h)-f(x_0)}{h}=\lim_{h\to0^+}\frac{f(x_0)-f(x_0-h)}{h},$$

$$f'(x_0)=\lim_{h\to0^-}\frac{f(x_0+h)-f(x_0)}{h}=\lim_{h\to0^-}\frac{f(x_0)-f(x_0-h)}{h}。$$

定理 1[12]　若 $f:\mathbf{R}\to L(R)$ 且 $f(t,\rho)=[\underline{f}(t,\rho),\bar{f}(t,\rho)]$，则 $f'(t,\rho)=[\underline{f}'(t,\rho),\bar{f}'(t,\rho)]$。

定义 3（Sumudu 变换）[14]　对任意 $f(t)\in L(R)$ 且 $-\tau_1<u<\tau_2$，则有

$$F(u)=S[f(t)]=\int_0^\infty\frac{1}{u}\mathrm{e}^{-(\frac{t}{u})}f(t)\mathrm{d}t。$$

定理 2[15]　若 C_1、C_2、C_3 均为大于 0 的常数，且对任意 $f_1(t)$、$f_2(t)$、$f(t)$ 分别具有 Sumudu 变换 $G_1(u)$、$G_2(u)$ 和 $G(u)$，则：

（ⅰ）$S[C_1f_1(t)+C_2f_2(t)]=C_1S[f_1(t)]+C_2[f_2(t)]=C_1G_1(u)+C_2G_2(u)$；

（ⅱ）$S[f(t)]=G(Cu)$；

（ⅲ）$\lim\limits_{t\to0}f(t)=f(0)=\lim\limits_{u\to0}G(u)$。

定义 4（模糊 Sumudu 变换）[14-16]　对任意 $f:\mathbf{R}\to L(R)$，则有

$$\boldsymbol{F}(u)=S[f(t,\rho)]=(S[\underline{f}(t,\rho)],S[\bar{f}(t,\rho)]。$$

定理 3[17]　若对 $\forall\,0<\rho\leqslant1$ 有 $f:\mathbf{R}\to L(R)$ 且 $f(x)=[\underline{f}(x,\rho),\bar{f}(x,\rho)]$，则：当 f 为第ⅰ种可微时有 $f'(x,\rho)=[\underline{f}'(x,\rho),\bar{f}'(x,\rho)]$；当 f 为第ⅱ种可微时有 $f'(x,\rho)=[\bar{f}'(x,\rho),\underline{f}'(x,\rho)]$。

定理 4[17]　若 $f:\mathbf{R}\to L(R)$ 是一个连续的带有模糊初值的函数，且 $F(u)=S[f(x,\rho)]$，$u>0$，则：当 f 为第ⅰ种可微时有 $S[f'(x,\rho)]=\dfrac{F(u)}{u}-\dfrac{f(0)}{u}$，当 f 为第ⅱ种可微时有 $S[f'(x,\rho)]=-\dfrac{f(0)}{u}-\dfrac{(-F(u))}{u}$。

定理 5[17]　若 $f:\mathbf{R}\to L(R)$ 是一个连续的带有模糊初值的函数，且 $F(u)=S[f(x,\rho)]$，则

$$S\left[\int_0^x f(t)\mathrm{d}t\right]=uF(u)。$$

2　Volterra 模糊积分方程的 Sumudu 分解法

现采用一种新的数值解析方法来求解 Volterra 积分方程

$$\varphi'(x)+\lambda\int_a^x K(x,t)\varphi(t)\mathrm{d}t=M(x)\varphi(x)+f(x)\tag{1}$$

的模糊数值解，初值条件为 $\varphi(0)=\alpha_0=(\underline{\alpha_0},\bar{\alpha_0})$。对方程(1)进行 Sumudu 变换可得

$$S[\varphi'(x)]+S[\lambda\int_a^x K(x,t)\varphi(t)\mathrm{d}t]=S[M(x)\varphi(x)]+S[f(x)]。\tag{2}$$

方程(2)整理可得

$$\frac{1}{u}S[\varphi(x)]-\frac{1}{u}\varphi(0)+\lambda uS[K(x,t)]S[\varphi(x)]=S[M(x)]S[\varphi(x)]+S[f(x)]。\tag{3}$$

方程(3)可写为

$$\frac{1}{u}S[\underline{\varphi}(x)]-\frac{1}{u}\underline{\varphi}(0)+\lambda uS[\underline{K(x,t)]S[\varphi(x)}]=S[\underline{M(x)]S[\varphi(x)}]+S[\underline{f}(x)],\tag{4}$$

$$\frac{1}{u}S[\bar{\varphi}(x)]-\frac{1}{u}\bar{\varphi}(0)+\lambda uS[\overline{K(x,t)]S[\varphi(x)}]=S[\overline{M(x)]S[\varphi(x)}]+S[\bar{f}(x)]。\tag{5}$$

由于 $\varphi(0)=(\underline{\alpha}_0,\overline{\alpha}_0)$，因此有等价方程

$$\frac{1}{u}S[\underline{\varphi}(x)]-\frac{1}{u}\underline{\alpha}_0+\lambda uS[\underline{K(x,t)]S[\varphi(x)}]=S[\underline{M(x)]S[\varphi(x)}]+S[\underline{f}(x)], \tag{6}$$

$$\frac{1}{u}S[\overline{\varphi}(x)]-\frac{1}{u}\overline{\alpha}_0+\lambda uS[\overline{K(x,t)]S[\varphi(x)}]=S[\overline{M(x)]S[\varphi(x)}]+S[\overline{f}(x)]。 \tag{7}$$

以下分 8 种情形对方程(6)(7)进行讨论：

①当 $\varphi(x)$、$K(x,t)$、$M(x)$ 均为正时，

$$S[\underline{K(x,t)]S[\varphi(x)}]=S[\underline{K}(x,t)]S[\underline{\varphi}(x)],S[\overline{K(x,t)]S[\varphi(x)}]=S[\overline{K}(x,t)]S[\overline{\varphi}(x)],$$

$$S[\underline{M(x)]S[\varphi(x)}]=S[\underline{M}(x)]S[\underline{\varphi}(x)],S[\overline{M(x)]S[\varphi(x)}]=S[\overline{M}(x)]S[\overline{\varphi}(x)]。$$

②当 $\varphi(x)$、$K(x,t)$、$M(x)$ 均为负时，

$$S[\underline{K(x,t)]S[\varphi(x)}]=S[\overline{K}(x,t)]S[\overline{\varphi}(x)],S[\overline{K(x,t)]S[\varphi(x)}]=S[\underline{K}(x,t)]S[\underline{\varphi}(x)],$$

$$S[\underline{M(x)]S[\varphi(x)}]=S[\overline{M}(x)]S[\overline{\varphi}(x)],S[\overline{M(x)]S[\varphi(x)}]=S[\underline{M}(x)]S[\underline{\varphi}(x)]。$$

③当 $\varphi(x)$ 为正，$K(x,t)$、$M(x)$ 均为负时，

$$S[\underline{K(x,t)]S[\varphi(x)}]=S[\underline{K}(x,t)]S[\overline{\varphi}(x)],S[\overline{K(x,t)]S[\varphi(x)}]=S[\overline{K}(x,t)]S[\underline{\varphi}(x)],$$

$$S[\underline{M(x)]S[\varphi(x)}]=S[\underline{M}(x)]S[\overline{\varphi}(x)],S[\overline{M(x)]S[\varphi(x)}]=S[\overline{M}(x)]S[\underline{\varphi}(x)]。$$

④当 $\varphi(x)$、$K(x,t)$ 均为正，$M(x)$ 为负时，

$$S[\underline{K(x,t)]S[\varphi(x)}]=S[\underline{K}(x,t)]S[\underline{\varphi}(x)],S[\overline{K(x,t)]S[\varphi(x)}]=S[\overline{K}(x,t)]S[\overline{\varphi}(x)],$$

$$S[\underline{M(x)]S[\varphi(x)}]=S[\underline{M}(x)]S[\overline{\varphi}(x)],S[\overline{M(x)]S[\varphi(x)}]=S[\overline{M}(x)]S[\underline{\varphi}(x)]。$$

⑤当 $\varphi(x)$、$M(x)$ 均为正，$K(x,t)$ 为负时

$$S[\underline{K(x,t)]S[\varphi(x)}]=S[\underline{K}(x,t)]S[\overline{\varphi}(x)],S[\overline{K(x,t)]S[\varphi(x)}]=S[\overline{K}(x,t)]S[\underline{\varphi}(x)],$$

$$S[\underline{M(x)]S[\varphi(x)}]=S[\underline{M}(x)]S[\underline{\varphi}(x)],S[\overline{M(x)]S[\varphi(x)}]=S[\overline{M}(x)]S[\overline{\varphi}(x)]。$$

⑥当 $\varphi(x)$、$K(x,t)$ 均为负，$M(x)$ 为正时，

$$S[\underline{K(x,t)]S[\varphi(x)}]=S[\overline{K}(x,t)]S[\overline{\varphi}(x)],S[\overline{K(x,t)]S[\varphi(x)}]=S[\underline{K}(x,t)]S[\underline{\varphi}(x)],$$

$$S[\underline{M(x)]S[\varphi(x)}]=S[\overline{M}(x)]S[\underline{\varphi}(x)],S[\overline{M(x)]S[\varphi(x)}]=S[\underline{M}(x)]S[\overline{\varphi}(x)]。$$

⑦当 $\varphi(x)$、$M(x)$ 均为负，$K(x,t)$ 为正时，

$$S[\underline{K(x,t)]S[\varphi(x)}]=S[\overline{K}(x,t)]S[\underline{\varphi}(x)],S[\overline{K(x,t)]S[\varphi(x)}]=S[\underline{K}(x,t)]S[\overline{\varphi}(x)],$$

$$S[\underline{M(x)]S[\varphi(x)}]=S[\overline{M}(x)]S[\overline{\varphi}(x)],S[\overline{M(x)]S[\varphi(x)}]=S[\underline{M}(x)]S[\underline{\varphi}(x)]。$$

⑧当 $\varphi(x)$ 为负，$K(x,t)$、$M(x)$ 均为正时，

$$S[\underline{K(x,t)]S[\varphi(x)}]=S[\overline{K}(x,t)]S[\underline{\varphi}(x)],S[\overline{K(x,t)]S[\varphi(x)}]=S[\underline{K}(x,t)]S[\overline{\varphi}(x)],$$

$$S[\underline{M(x)]S[\varphi(x)}]=S[\overline{M}(x)]S[\underline{\varphi}(x)],S[\overline{M(x)]S[\varphi(x)}]=S[\underline{M}(x)]S[\overline{\varphi}(x)]。$$

当方程(6)(7)为第①种情况（其他类似）时，

$$S[\underline{\varphi}(x)]-\underline{\alpha}_0+\lambda u^2S[\underline{K}(x,t)]S[\underline{\varphi}(x)]=uS[\underline{M}(x)]S[\underline{\varphi}(x)]+S[\underline{f}(x)], \tag{8}$$

$$S[\overline{\varphi}(x)]-\overline{\alpha}_0+\lambda u^2S[\overline{K}(x,t)]S[\overline{\varphi}(x)]=uS[\overline{M}(x)]S[\overline{\varphi}(x)]+uS[\overline{f}(x)]。 \tag{9}$$

由方程(8)(9)可得

$$S[\underline{\varphi}(x)]+\lambda u^2S[\underline{K}(x,t)]S[\underline{\varphi}(x)]-uS[\underline{M}(x)]S[\underline{\varphi}(x)]=uS[\underline{f}(x)]+\underline{\alpha}_0,$$

$$S[\overline{\varphi}(x)]+\lambda u^2S[\overline{K}(x,t)]S[\overline{\varphi}(x)]-uS[\overline{M}(x)]S[\overline{\varphi}(x)]=uS[\overline{f}(x)]+\overline{\alpha}_0。$$

即有

$$S[\underline{\varphi}(x)]=\frac{uS[\underline{f}(x)]+\underline{\alpha_0}}{1+\lambda u^2S[\underline{K}(x,t)]-uS[\underline{M}(x)]},S[\overline{\varphi}(x)]=\frac{uS[\overline{f}(x)]+\overline{\alpha_0}}{1+\lambda u^2S[\overline{K}(x,t)]-uS[\overline{M}(x)]},$$

利用 Sumudu 变换可得 $\underline{\varphi}(x)$ 和 $\overline{\varphi}(x)$。下面采用分解法求解 $\underline{\varphi}(x)$ 和 $\overline{\varphi}(x)$。

$$\sum_{i=0}^{\infty}\underline{\varphi_i}(x)=\underline{\varphi_0}(x)+\underline{\varphi_1}(x)+\cdots+\underline{\varphi_n}(x)+\cdots,\ \sum_{i=0}^{\infty}\overline{\varphi_i}(x)=\overline{\varphi_0}(x)+\overline{\varphi_1}(x)+\cdots+\overline{\varphi_n}(x)+\cdots,$$

则

$$S[\underline{\varphi_0}(x)]=uS[\underline{f(x)}]+\underline{\alpha_0},$$
$$S[\underline{\varphi_1}(x)]=uS[\underline{M}(x)]S[\underline{\varphi_0}(x)]-\lambda u^2S[\underline{K}(x,t)]S[\underline{\varphi_0}(x)],$$
$$S[\underline{\varphi_2}(x)]=uS[\underline{M}(x)]S[\underline{\varphi_1}(x)]-\lambda u^2S[\underline{K}(x,t)]S[\underline{\varphi_1}(x)],$$
$$\cdots\cdots$$
$$S[\underline{\varphi_n}(x)]=uS[\underline{M}(x)]S[\underline{\varphi_{n-1}}(x)]-\lambda u^2S[\underline{K}(x,t)]S[\underline{\varphi_{n-1}}(x)]。$$

同样可得

$$S[\overline{\varphi_0}(x)]=uS[\overline{f}(x)]+\overline{\alpha_0},$$
$$S[\overline{\varphi_1}(x)]=uS[\overline{M}(x)]S[\overline{\varphi_0}(x)]-\lambda u^2S[\overline{K}(x,t)]S[\overline{\varphi_0}(x)],$$
$$S[\overline{\varphi_2}(x)]=uS[\overline{M}(x)]S[\overline{\varphi_1}(x)]-\lambda u^2S[\overline{K}(x,t)]S[\overline{\varphi_1}(x)],$$
$$\cdots\cdots$$
$$S[\overline{\varphi_n}(x)]=uS[\overline{M}(x)]S[\overline{\varphi_{n-1}}(x)]-\lambda u^2S[\overline{K}(x,t)]S[\overline{\varphi_{n-1}}(x)]。$$

由上式可得

$$S[\sum_{i=0}^{\infty}\underline{\varphi_i}(x)]-\underline{\alpha_0}=uS[\underline{f}(x)]+uS[\underline{M}(x)]S[\sum_{j=1}^{\infty}\underline{\varphi_j}(x)]-\lambda u^2S[\underline{K}(x,t)]S[\sum_{j=1}^{\infty}\underline{\varphi_j}(x)],$$
$$S[\sum_{i=0}^{\infty}\overline{\varphi_i}(x)]-\overline{\alpha_0}=uS[\overline{f}(x)]+uS[\overline{M}(x)]S[\sum_{j=1}^{\infty}\overline{\varphi_j}(x)]-\lambda u^2S[\overline{K}(x,t)]S[\sum_{j=1}^{\infty}\overline{\varphi_j}(x)]。$$

3　例题

为了验证上述方法的可行性,运用该方法求解一个实例。

例 1　$\varphi'(x)+\int_0^x\varphi(t)\mathrm{d}t=f(x),0\leqslant t\leqslant x,0\leqslant\rho\leqslant 1$,其中,

$$\lambda=1,\varphi(0)=(\rho+1,3-\rho),f(x)=(\rho^2+\rho,5-\rho),K(x,t)=1。$$

由上述方法可知

$$\underline{\varphi'}(x)=\underline{f}(x)-\int_0^x\underline{\varphi}(t)\mathrm{d}t,\ \overline{\varphi'}(x)=\overline{f}(x)-\int_0^x\overline{\varphi}(t)\mathrm{d}t。$$

即

$$\underline{\varphi'}(x)=(\rho^2+\rho)-\int_0^x\underline{\varphi}(t)\mathrm{d}t,\overline{\varphi'}(x)=(5-\rho)-\int_0^x\overline{\varphi}(t)\mathrm{d}t。$$

利用模糊 Sumudu 变换可得

$$\underline{\varphi'}(x)=S^{-1}[u(\rho^2+\rho)]-S^{-1}[u^2S[\underline{\varphi}(x)]],\overline{\varphi'}(x)=S^{-1}[u(5-\rho)]-S^{-1}[u^2S[\overline{\varphi}(x)]]。$$

解得

$$\underline{\varphi_0}=x(\rho^2+\rho),\underline{\varphi_1}(x)=-\frac{x^3}{3!}(\rho^2+\rho),$$

类似可得

$$\underline{\varphi_2}(x)=\frac{x^5}{5!}(\rho^2+\rho),\underline{\varphi_3}(x)=-\frac{x^7}{7!}(\rho^2+\rho),\cdots\cdots$$

$$\overline{\varphi_0}(x)=x(5-\rho),\overline{\varphi_1}(x)=-\frac{x^3}{3!}(5-\rho),\overline{\varphi_2}(x)=\frac{x^5}{5!}(5-\rho),\overline{\varphi_3}(x)=-\frac{x^7}{7!}(5-\rho),\cdots\cdots$$

则

$$\underline{\varphi}(x)=\sum_{i=1}^{\infty}\underline{\varphi_i}(x)=x(\rho^2+\rho)-\frac{x^3}{3!}(\rho^2+\rho)+\cdots,$$

$$\overline{\varphi}(x)=\sum_{i=0}^{\infty}\overline{\varphi_i}(x)=x(5-\rho)-\frac{x^3}{3!}(5-\rho)+\cdots$$

或

$$\underline{\varphi}(x)=(\rho^2+\rho)\sin x,\overline{\varphi}(x)=(5-\rho)\sin x,0\leqslant\rho\leqslant 1。$$

即可求出数值解如图 1 所示。

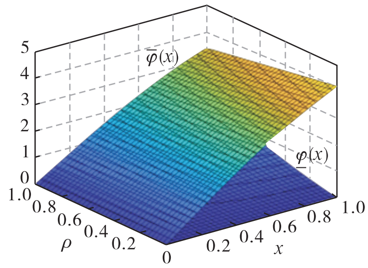

图 1 例 1 的数值解

4 结论

本文应用 Sumudu 分解法对 Volterra 模糊积分方程的数值解进行了深入探讨。将问题通过参数化方式表述,并将其转化为两个等效的常规积分-微分方程来求解。数值实验结果验证了 Sumudu 分解法在解决此类问题上的有效性与可靠性。可见,Sumudu 分解法是解决 Volterra 模糊积分方程数值问题的一种有效手段。然而,Sumudu 分解法在解决分数阶模糊积分方程的数值解方面的适用性仍有待进一步研究。

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Numerical solution of Volterra fuzzy integral equation

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Abstract：The Sumudu decomposition method was used to numerically solve the Volterra fuzzy integral equation，and the effectiveness and practicality of the method were verified through detailed numerical examples.

Keywords：Volterra integral equation；Sumudu decomposition method；fuzzy integral equation；numerical solution；Sumudu transformation

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