【微分方程与动力系统研究】

具有时变时滞的复值 BAM 神经网络的概周期解

赵莉莉

（云南大学 数学与统计学院，云南 昆明 650091）

摘　要：研究了一类具有时变时滞的复值 BAM 神经网络，利用 Banach 空间中的不动点定理、实数集上的指数二分性，以及若干微分不等式技巧，获得了该类复值神经网络的概周期解存在、唯一，以及一致稳定的充分条件。最后，通过实例验证了所得结果的有效性与可行性。

关键词：复值神经网络；BAM 神经网络；概周期解；不动点定理；一致稳定性

中图分类号：O 175　　**文献标识码**：A　　DOI：10.13486/j.cnki.1673-2618.2024.02.008

0　引言

神经网络是在许多学科的基础上发展起来的综合性、交叉性很强的学科，它能够从人脑的神经系统结构出发，研究大脑的工作机制，进而揭示人工智能的本质。自神经网络的数学模型建立后，研究人员对实值神经网络的研究取得了大量的成果[1-5]。虽然实值神经网络在自动控制、模式识别、图像处理、医疗卫生等领域得到广泛应用，但其无法直接处理复数数据。因此，作为实值神经网络的推广，复值神经网络应运而生，它解决了一些实值神经网络不能解决的问题[6]。近年来，对复值神经网络的研究主要集中在探讨其平衡点、周期解，以及反周期解的存在性、各类稳定性、无源性，以及耗散性，也取得了大量的研究成果[7-11]。

相较于周期现象，概周期现象是频繁的、经常发生的，而且概周期解的存在性与稳定性对描述动力系统的动力学行为是非常重要的，然而少有文献研究复值神经网络概周期解的相关问题。文献[12]研究了一类具有时变时滞的复值单层神经网络概周期解的存在性与稳定性。但是，与传统的单层神经网络模型相比，采用异联想原理，实现网络状态在两层神经元之间来回传递的 BAM 神经网络不仅有理论意义，还在联想记忆、复杂优化问题、模式识别、信号处理以及自动控制等领域有强大的应用功能。

基于此，本文利用 Banach 空间中的不动点定理和指数型二分性，探讨具有时变时滞的复值 BAM 神经网络

$$\begin{cases} u_i'(t) = -c_i(t)u_i(t) + \sum_{j=1}^{m} a_{ij}(t) f_j(v_j(t)) + \sum_{j=1}^{m} \alpha_{ij}(t) h_j(v_j(t-\tau_{ij}(t))) + I_i(t), \\ v_j'(t) = -d_j(t)v_j(t) + \sum_{i=1}^{n} b_{ji}(t) g_i(u_i(t)) + \sum_{i=1}^{n} l_{ji}(t) \eta_i(u_i(t-\omega_{ji}(t))) + J_j(t) \end{cases} \tag{1}$$

的概周期解的存在性与稳定性问题。其中：$i=1,2,\cdots,n$，$j=1,2,\cdots,m$；$u_i(t)$，$v_j(t)\in\mathbf{C}$，分别表示第 I 层第 i 个神经元、第 J 层第 j 个神经元在 t 时刻的状态；$c_i(t)$，$d_j(t)>0$ 表示自反馈连接权重；$a_{ij}(t)$，$\alpha_{ij}(t)$，

收稿日期：2023-07-23

基金项目：云南省教育厅自然科学基金项目（2020J0020）；云南大学教育教学改革基金项目（2023Y22）

作者简介：赵莉莉（1979—），女，白族，云南大理人，讲师，博士，主要从事非线性微分方程与动力系统研究。

E-mail：llzhao@ynu.edu.cn

$b_{ji}(t),l_{ji}(t)\in\mathbf{C}$ 表示神经元之间的连接权重；$I_i(t),J_j(t)\in\mathbf{C}$ 分别表示第 I 层第 i 个神经元、第 J 层第 j 个神经元在 t 时刻的外部输入；$f_j,h_j,g_i,\eta_i\in\mathbf{C}$ 表示神经元的激活函数；$\tau_{ij}(t),\omega_{ji}(t)$表示神经元的传输时滞，且满足 $0\leqslant\tau_{ij}(t),\omega_{ij}(t)\leqslant\kappa,\kappa>0$ 为常数。

1　概周期实值系统与复值系统

定义 1　对于复值函数 $u(t)=u^{\mathrm{R}}(t)+\mathrm{i}u^{\mathrm{I}}(t)$，其中 $u^{\mathrm{R}}=\mathrm{Re}(u(t)),u^{\mathrm{I}}(t)=\mathrm{Im}(u(t))$，若 $u^{\mathrm{R}}(t)$与 $u^{\mathrm{I}}(t)$均为概周期函数，则称 $u(t)$为概周期复值函数。

定义 2　对于矩阵函数 $\boldsymbol{A}(t)=(a_{ij}(t))$，若每一个 $a_{ij}(t)$都是概周期函数，则称 $\boldsymbol{A}(t)$为概周期矩阵函数；对于向量函数 $\boldsymbol{u}(t)=(u_1(t),u_2(t),\cdots,u_n(t))^{\mathrm{T}}$，若每一个 $u_i(t)$都是概周期函数，则称 $\boldsymbol{u}(t)$为概周期向量函数。

考虑概周期实值系统

$$\boldsymbol{x}'(t)=\boldsymbol{A}(t)\boldsymbol{x}(t) \tag{2}$$

以及

$$\boldsymbol{x}'(t)=\boldsymbol{A}(t)\boldsymbol{x}(t)+\boldsymbol{f}(t), \tag{3}$$

其中 $\boldsymbol{A}(t)$是关于 t 的概周期矩阵函数，$\boldsymbol{f}(t)$是关于 t 的概周期向量函数。

定义 3[13-14]　如果存在投影算子 $\boldsymbol{P}$ 以及正常数 M,α,β 使得系统(2)的基解矩阵 $\boldsymbol{X}(t)$满足

$$\|\boldsymbol{X}(t)\boldsymbol{P}\boldsymbol{X}^{-1}(s)\|\leqslant M\mathrm{e}^{-\alpha(t-s)}(t\geqslant s),\ \|\boldsymbol{X}(t)(I-\boldsymbol{P})\boldsymbol{X}^{-1}(s)\|\leqslant M\mathrm{e}^{-\beta(s-t)}(s\geqslant t),$$

则称系统(2)在实数集上满足指数型二分性。

引理 1[13-14]　若系统(2) 在实数集上满足指数型二分性，则系统(3) 存在唯一的概周期解

$$\boldsymbol{x}(t)=\int_{-\infty}^{t}\boldsymbol{X}(t)\boldsymbol{P}\boldsymbol{X}^{-1}(s)\boldsymbol{f}(s)\mathrm{d}s-\int_{t}^{+\infty}\boldsymbol{X}(t)(I-\boldsymbol{P})\boldsymbol{X}^{-1}(s)\boldsymbol{f}(s)\mathrm{d}s。$$

引理 2[15]　若概周期函数 $a(t)$ 的平均值 $M[a(t)]=\lim\limits_{t-s\to+\infty}\dfrac{1}{t-s}\displaystyle\int_s^t a(\tau)\mathrm{d}\tau<0$，则存在正常数 α_1,α_2 使得 $\exp(\int_s^t a(\tau)\mathrm{d}\tau)\leqslant\alpha_1\exp(-\alpha_2(t-s))(t\geqslant s)$。

为了探讨神经网络(1) 概周期解的存在性与稳定性，做如下假设：

(C_1) $u_i=u_i^{\mathrm{R}}+\mathrm{i}u_i^{\mathrm{I}},v_j=v_j^{\mathrm{R}}+\mathrm{i}v_j^{\mathrm{I}},u_i^{\mathrm{R}},u_i^{\mathrm{I}},v_j^{\mathrm{R}},v_j^{\mathrm{I}}\in\mathbf{R}$，那么 $f_j(v_j),h_j(v_j),g_i(u_i),\eta_i(u_i)$ 可以表示为

$$f_j(v_j)=f_j^{\mathrm{R}}(v_j^{\mathrm{R}},v_j^{\mathrm{I}})+\mathrm{i}f_j^{\mathrm{I}}(v_j^{\mathrm{R}},v_j^{\mathrm{I}}),h_j(v_j)=h_j^{\mathrm{R}}(v_j^{\mathrm{R}},v_j^{\mathrm{I}})+\mathrm{i}h_j^{\mathrm{I}}(v_j^{\mathrm{R}},v_j^{\mathrm{I}}),$$

$$g_i(u_i)=g_i^{\mathrm{R}}(u_i^{\mathrm{R}},u_i^{\mathrm{I}})+\mathrm{i}g_i^{\mathrm{I}}(u_i^{\mathrm{R}},u_i^{\mathrm{I}}),\eta_i(u_i)=\eta_i^{\mathrm{R}}(u_i^{\mathrm{R}},u_i^{\mathrm{I}})+\mathrm{i}\eta_i^{\mathrm{I}}(u_i^{\mathrm{R}},u_i^{\mathrm{I}}),$$

其中 $f_j^{\mathrm{R}},f_j^{\mathrm{I}},h_j^{\mathrm{R}},h_j^{\mathrm{I}},g_i^{\mathrm{R}},g_i^{\mathrm{I}},\eta_i^{\mathrm{R}},\eta_i^{\mathrm{I}}:\mathbf{R}^2\to\mathbf{R}$。

(C_2) $a_{ij}(t)=a_{ij}^{\mathrm{R}}(t)+\mathrm{i}a_{ij}^{\mathrm{I}}(t),\alpha_{ij}(t)=\alpha_{ij}^{\mathrm{R}}(t)+\mathrm{i}\alpha_{ij}^{\mathrm{I}}(t),b_{ji}(t)=b_{ji}^{\mathrm{R}}(t)+\mathrm{i}b_{ji}^{\mathrm{I}}(t),l_{ji}(t)=l_{ji}^{\mathrm{R}}(t)+\mathrm{i}l_{ji}^{\mathrm{I}}(t)$，$I_i(t)=I_i^{\mathrm{R}}(t)+\mathrm{i}I_i^{\mathrm{I}}(t),J_j(t)=J_j^{\mathrm{R}}(t)+\mathrm{i}J_j^{\mathrm{I}}(t)$，其中 $a_{ij}^{\mathrm{R}},a_{ij}^{\mathrm{I}},\alpha_{ij}^{\mathrm{R}},\alpha_{ij}^{\mathrm{I}},b_{ji}^{\mathrm{R}},b_{ji}^{\mathrm{I}},l_{ji}^{\mathrm{R}},l_{ji}^{\mathrm{I}},I_i^{\mathrm{R}},I_i^{\mathrm{I}},J_j^{\mathrm{R}},J_j^{\mathrm{I}}:\mathbf{R}\to\mathbf{R}$。

由假设(C_1)(C_2)，系统(1) 可以化为实值系统

$$(u_i^{\mathrm{R}})'(t)=-c_i(t)u_i^{\mathrm{R}}(t)+\sum_{j=1}^{m}(a_{ij}^{\mathrm{R}}(t)f_j^{\mathrm{R}}(v_j^{\mathrm{R}}(t),v_j^{\mathrm{I}}(t))-a_{ij}^{\mathrm{I}}(t)f_j^{\mathrm{I}}(v_j^{\mathrm{R}}(t),v_j^{\mathrm{I}}(t)))+$$

$$\sum_{j=1}^{m}(\alpha_{ij}^{\mathrm{R}}(t)h_j^{\mathrm{R}}(v_j^{\mathrm{R}}(t-\tau_{ij}(t)),v_j^{\mathrm{I}}(t-\tau_{ij}(t)))-\alpha_{ij}^{\mathrm{I}}(t)h_j^{\mathrm{I}}(v_j^{\mathrm{R}}(t-\tau_{ij}(t)),v_j^{\mathrm{I}}(t-\tau_{ij}(t))))+I_i^{\mathrm{R}}(t), \tag{4}$$

$$(u_i^{\mathrm{I}})'(t)=-c_i(t)u_i^{\mathrm{I}}(t)+\sum_{j=1}^{m}(a_{ij}^{\mathrm{R}}(t)f_j^{\mathrm{I}}(v_j^{\mathrm{R}}(t),v_j^{\mathrm{I}}(t))+a_{ij}^{\mathrm{I}}(t)f_j^{\mathrm{R}}(v_j^{\mathrm{R}}(t),v_j^{\mathrm{I}}(t)))+$$

$$\sum_{j=1}^{m}(\alpha_{ij}^{\mathrm{R}}(t)h_j^{\mathrm{I}}(v_j^{\mathrm{R}}(t-\tau_{ij}(t)),v_j^{\mathrm{I}}(t-\tau_{ij}(t)))+\alpha_{ij}^{\mathrm{I}}(t)h_j^{\mathrm{R}}(v_j^{\mathrm{R}}(t-\tau_{ij}(t)),v_j^{\mathrm{I}}(t-\tau_{ij}(t))))+I_i^{\mathrm{I}}(t), \tag{5}$$

$$(v_j^{\mathrm{R}})'(t)=-d_j(t)v_j^{\mathrm{R}}(t)+\sum_{i=1}^{n}(b_{ji}^{\mathrm{R}}(t)g_i^{\mathrm{R}}(u_i^{\mathrm{R}}(t),u_i^{\mathrm{I}}(t))-b_{ji}^{\mathrm{I}}(t)g_i^{\mathrm{I}}(u_i^{\mathrm{R}}(t),u_j^{\mathrm{I}}(t)))+$$

$$\sum_{j=1}^{m}(l_{ji}^{R}(t)\eta_i^{R}(u_i^{R}(t-\omega_{ji}(t)),u_i^{I}(t-\omega_{ji}(t)))-l_{ji}^{I}(t)\eta_i^{I}(u_i^{R}(t-\omega_{ji}(t)),u_i^{I}(t-\omega_{ji}(t))))+J_j^{R}(t),\quad(6)$$

$$(v_j^{I})'(t)=-d_j(t)v_j^{I}(t)+\sum_{i=1}^{n}(b_{ji}^{R}(t)g_i^{I}(u_i^{R}(t),u_i^{I}(t))+b_{ji}^{I}(t)g_i^{R}(u_i^{R}(t),u_j^{I}(t)))+$$

$$\sum_{i=1}^{n}(l_{ji}^{R}(t)\eta_i^{I}(u_i^{R}(t-\omega_{ji}(t)),u_i^{I}(t-\omega_{ji}(t)))+l_{ji}^{I}(t)\eta_i^{R}(u_i^{R}(t-\omega_{ji}(t)),u_i^{I}(t-\omega_{ji}(t))))+J_j^{I}(t)。\quad(7)$$

其中：$i=1,2,\cdots,n$；$j=1,2,\cdots,m$。如无特殊说明，下文 i,j 取值与此相同。因此，对复值神经网络(1) 的概周期解的研究，可以转化为对实值神经网络(4)（包含式(4) ～ (7) 四个式子）的概周期解的研究。

系统(4) 的初值条件为

$$u_i^{R}(s)=\varphi_i^{R}(s),u_i^{I}(s)=\varphi_i^{I}(s),v_j^{R}(s)=\varphi_{n+j}^{R}(s),v_j^{I}(s)=\varphi_{n+j}(s),s\in[t_0-\kappa,t_0],t_0\geqslant 0,$$

其中

$$\boldsymbol{\Phi}(s)=(\varphi_1^{R}(s),\varphi_1^{I}(s),\cdots,\varphi_n^{R}(s),\varphi_n^{I}(s),\varphi_{n+1}^{R}(s),\varphi_{n+1}^{I}(s),\cdots,\varphi_{n+m}^{R}(s),\varphi_{n+m}^{I}(s))^{T}:[t_0-\kappa,t_0]\to\mathbf{R}^{n+m}$$

为连续的向量函数。

定义 4 具有初值 $\boldsymbol{\Phi}$ 的系统(4) 的解

$$\boldsymbol{U}(t)=(u_1^{R}(t),u_1^{I}(t),\cdots,u_n^{R}(t),u_n^{I}(t),v_1^{R}(t),\cdots,v_m^{R}(t),v_m^{I}(t))^{T}(t\geqslant t_0)$$

称为一致稳定的，如果对于任意的 $\varepsilon>0$ 和 $t_0\geqslant 0$，存在与 t_0 无关的正常数 $\delta=\delta(\varepsilon)$，使得任意的具有初值 Λ 的系统(4) 的解

$$X(t)=(x_1^{R}(t),x_1^{I}(t),\cdots,x_n^{R}(t),x_n^{I}(t),y_1^{R}(t),y_1^{I}(t),\cdots,y_m^{R}(t),y_m^{I}(t))^{T}$$

只要 $\|\boldsymbol{\Phi}-\boldsymbol{\Psi}\|<\delta$，就有 $\|\boldsymbol{U}(t)-\boldsymbol{X}(t)\|<\varepsilon,(t\geqslant t_0)$，其中

$$\|\boldsymbol{\Phi}-\boldsymbol{\Lambda}\|=\max_{1\leqslant k\leqslant n+m}\{\max\{\sup_{s\in[t_0-\kappa,t_0]}|\varphi_\kappa^{R}(s)-\Lambda_\kappa^{R}(s)|,\sup_{s\in[t_0-\kappa,t_0]}|\varphi_\kappa^{I}(s)-\Lambda_\kappa^{I}(s)|\}\},$$

$$\|\boldsymbol{U}(t)-\boldsymbol{X}(t)\|=\max\{\max_{1\leqslant i\leqslant n}|u_i^{R}(t)-x_i^{R}(t)|,\max_{1\leqslant i\leqslant n}|u_i^{I}(t)-x_i^{I}(t)|,$$

$$\max_{1\leqslant j\leqslant m}|v_j^{R}(t)-y_j^{R}(t)|,\max_{1\leqslant j\leqslant m}|v_j^{I}(t)-y_j^{I}(t)|\}。$$

2 概周期解的存在性与稳定性

对于实值神经网络(4)，还需要做如下假设

(C$_3$) $c_i(t),d_j(t),a_{ij}^{R}(t),a_{ij}^{I}(t),\alpha_{ij}^{R}(t),\alpha_{ij}^{I},b_{ji}^{R}(t),b_{ji}^{I}(t),l_{ji}^{R}(t),I_{ji}^{I}(t),I_i^{R}(t),I_i^{I}(t),J_j^{R}(t),J_j^{I}(t)$ 都是概周期函数，且 $M[c_i(t)]>0,M[d_j(t)]>0,i=1,2,\cdots,n,j=1,2,\cdots,m$。

(C$_4$) 存在函数 $f_{1j}^{\varpi}(x_j^{R},x_j^{I}),f_{2j}^{\varpi}(x_j^{R},x_j^{I}),h_{1j}^{\varpi}(x_j^{R},x_j^{I}),h_{2j}^{\varpi}(x_j^{R},x_j^{I}),g_{1i}^{\varpi}(y_i^{R},y_i^{I}),g_{2i}^{\varpi}(y_i^{R},y_i^{I}),\eta_{1i}^{\varpi}(y_i^{R},y_i^{I}),$ $\eta_{2i}^{\varpi}(y_i^{R},y_i^{I})$，以及常数 $M_{2j}^{\varpi},N_{2j}^{\varpi},\Pi_{2i}^{\varpi},\Gamma_{2i}^{\varpi},p_{1j}^{\varpi},q_{1j}^{\varpi},p_{2j}^{\varpi},q_{2j}^{\varpi},\bar{p}_{1j}^{\varpi},\bar{q}_{1j}^{\varpi},\bar{p}_{2j}^{\varpi},\bar{q}_{2j}^{\varpi},\tilde{p}_{1i}^{\varpi},\tilde{q}_{1i}^{\varpi},\tilde{p}_{2i}^{\varpi},\tilde{q}_{2i}^{\varpi},\hat{p}_{1i}^{\varpi},\hat{q}_{1i}^{\varpi},\hat{p}_{2i}^{\varpi},\hat{q}_{2i}^{\varpi}$，使得对任意的$(x_j^{R},x_j^{I}),(z_j^{R},z_j^{I}),(y_i^{R},y_i^{I}),(u_i^{R},u_i^{I})\in\mathbf{R}^2$，有

$$f_j^{\varpi}(x_j^{R},x_j^{I})=f_{1j}^{\varpi}(x_j^{R},x_j^{I})f_{2j}^{\varpi}(x_j^{R},x_j^{I}),f_{1j}^{\varpi}(0,0)=0,|f_{2j}^{\varpi}(x_j^{R},x_j^{I})|\leqslant M_{2j}^{\varpi},$$

$$|f_{1j}^{\varpi}(x_j^{R},x_j^{I})-f_{1j}^{\varpi}(z_j^{R},z_j^{I})|\leqslant p_{1j}^{\varpi}|x_j^{R}-z_j^{R}|+q_{1j}^{\varpi}|x_j^{I}-z_j^{I}|,$$

$$|f_{2j}^{\varpi}(x_j^{R},x_j^{I})-f_{2j}^{\varpi}(z_j^{R},z_j^{I})|\leqslant p_{2j}^{\varpi}|x_j^{R}-z_j^{R}|+q_{2j}^{\varpi}|x_j^{I}-z_j^{I}|,$$

$$h_j^{\varpi}(x_j^{R},x_j^{I})=h_{1j}^{\varpi}(x_j^{R},x_j^{I})h_{2j}^{\varpi}(x_j^{R},x_j^{I}),h_{1j}^{\varpi}(0,0)=0,|h_{2j}^{\varpi}(x_j^{R},x_j^{I})|\leqslant N_{2j}^{\varpi},$$

$$|h_{1j}^{\varpi}(x_j^{R},x_j^{I})-h_{1j}^{\varpi}(z_j^{R},z_j^{I})|\leqslant \bar{p}_{1j}^{\varpi}|x_j^{R}-z_j^{R}|+\bar{q}_{1j}^{\varpi}|x_j^{I}-z_j^{I}|,$$

$$|h_{2j}^{\varpi}(x_j^{R},x_j^{I})-h_{2j}^{\varpi}(z_j^{R},z_j^{I})|\leqslant \bar{p}_{2j}^{\varpi}|x_j^{R}-z_j^{R}|+\bar{q}_{2j}^{\varpi}|x_j^{I}-z_j^{I}|,$$

$$g_i^{\varpi}(y_i^{R},y_i^{I})=g_{1j}^{\varpi}(y_i^{R},y_i^{I})g_{2i}^{\varpi}(y_i^{R},y_i^{I}),g_{1i}^{\varpi}(0,0)=0,|g_{2i}^{\varpi}(y_i^{R},y_i^{I})|\leqslant \Pi_{2i}^{\varpi},$$

$$|g_{1i}^{\varpi}(y_i^{R},y_i^{I})-g_{1i}^{\varpi}(u_i^{R},u_i^{I})|\leqslant \tilde{p}_{1i}^{\varpi}|y_i^{R}-u_i^{R}|+\tilde{q}_{1i}^{\varpi}|y_i^{I}-u_i^{I}|,$$

$$|g_{2i}^{\varpi}(y_i^{R},y_i^{I})-g_{2i}^{\varpi}(u_i^{R},u_i^{I})|\leqslant \tilde{p}_{2i}^{\varpi}|y_i^{R}-u_i^{R}|+\tilde{q}_{2i}^{\varpi}|y_i^{I}-u_i^{I}|,$$

$$\eta_i^{\tilde{\omega}}(y_i^R,y_i^I)=\eta_{1i}^{\tilde{\omega}}(y_i^R,y_i^I)\eta_{2i}^{\tilde{\omega}}(y_i^R,y_i^I),\eta_{1i}^{\tilde{\omega}}(0,0),\ |\eta_{2i}^{\tilde{\omega}}(y_i^R,y_i^I)|\leqslant \Gamma_{2i}^{\tilde{\omega}},$$
$$|\eta_{1i}^{\tilde{\omega}}(y_i^R,y_i^I)-\eta_{1i}^{\tilde{\omega}}(u_i^R,u_i^I)|\leqslant \hat{p}_{1i}^{\tilde{\omega}}|y_i^R-u_i^R|+\hat{q}_{1i}^{\tilde{\omega}}|y_i^I-u_i^I|,$$
$$|\eta_{2i}^{\tilde{\omega}}(y_i^R,y_i^I)-\eta_{2i}^{\tilde{\omega}}(u_i^R,u_i^I)|\leqslant \hat{p}_{2i}^{\tilde{\omega}}|y_i^R-u_i^R|+\hat{q}_{2i}^{\tilde{\omega}}|y_i^I-u_i^I|,$$
其中，$\tilde{\omega}\in\{R,I\}$。

(C_5) $\lambda=\max\{\max\limits_{1\leqslant i\leqslant n}\{\frac{\mu_i^R}{c_i},\frac{\mu_i^I}{c_i}\},\max\limits_{1\leqslant j\leqslant m}\{\frac{\vartheta_j^R}{d_j},\frac{\vartheta_j^I}{d_j}\}\}<1$，其中 $c_i=\inf\limits_{t\in\mathbf{R}}c_i(t)>0,d_j=\inf\limits_{t\in\mathbf{R}}d_j(t)>0$，

$$\mu_i^R=\sum_{j=1}^{m}[\bar{a}_{ij}^R(p_{1j}^R+q_{1j}^R)(M_{2j}^R+(p_{2j}^R+q_{2j}^R)\frac{\beta}{1-\gamma})+\bar{a}_{ij}^I(p_{1j}^I+q_{1j}^I)(M_{2j}^I+(p_{2j}^I+q_{2j}^I)\frac{\beta}{1-\gamma})+$$
$$\bar{\alpha}_{ij}^R(\bar{p}_{1j}^R+\bar{q}_{1j}^R)(N_{2j}^R+(\bar{p}_{2j}^R+\bar{q}_{2j}^R)\frac{\beta}{1-\gamma})+\bar{\alpha}_{ij}^I(\bar{p}_{1j}^I+\bar{q}_{1j}^I)(N_{2j}^I+(\bar{p}_{2j}^I+\bar{q}_{2j}^I)\frac{\beta}{1-\gamma})],$$

$$\mu_i^I=\sum_{j=1}^{m}[\bar{a}_{ij}^R(p_{1j}^I+q_{1j}^I)(M_{2j}^I+(p_{2j}^I+q_{2j}^I)\frac{\beta}{1-\gamma})+\bar{a}_{ij}^I(p_{1j}^R+q_{1j}^R)(M_{2j}^R+(p_{2j}^R+q_{2j}^R)\frac{\beta}{1-\gamma})+$$
$$\bar{\alpha}_{ij}^R(\bar{p}_{1j}^I+\bar{q}_{1j}^I)(N_{2j}^I+(\bar{p}_{2j}^I+\bar{q}_{2j}^I)\frac{\beta}{1-\gamma})+\bar{\alpha}_{ij}^I(\bar{p}_{1j}^R+\bar{q}_{1j}^R)(N_{2j}^R+(\bar{p}_{2j}^R+\bar{q}_{2j}^R)\frac{\beta}{1-\gamma})],$$

$$\vartheta_j^R=\sum_{i=1}^{n}[\bar{b}_{ji}^R(\tilde{p}_{1i}^R+\tilde{q}_{1i}^R)(\Pi_{2i}^R+(\tilde{p}_{2i}^R+\tilde{q}_{2i}^R)\frac{\beta}{1-\gamma})+\bar{b}_{ji}^I(\tilde{p}_{1i}^I+\tilde{q}_{1i}^I)(\Pi_{2i}^I+(\tilde{p}_{2i}^I+\tilde{q}_{2i}^I)\frac{\beta}{1-\gamma})+$$
$$\bar{l}_{ji}^R(\hat{p}_{1i}^R+\hat{q}_{1i}^R)(\Gamma_{2i}^R+(\hat{p}_{2i}^R+\hat{q}_{2i}^R)\frac{\beta}{1-\gamma})+\bar{l}_{ji}^I(\hat{p}_{1i}^I+\hat{q}_{1i}^I)(\Gamma_{2i}^I+(\hat{p}_{2i}^I+\hat{q}_{2i}^I)\frac{\beta}{1-\gamma})],$$

$$\vartheta_j^I=\sum_{i=1}^{m}[\bar{b}_{ji}^R(\tilde{p}_{1i}^I+\tilde{q}_{1i}^I)(\Pi_{2i}^I+(\tilde{p}_{2i}^I+\tilde{q}_{2i}^I)\frac{\beta}{1-\gamma})+\bar{b}_{ji}^I(\tilde{p}_{1i}^R+\tilde{q}_{1i}^R)(\Pi_{2i}^R+(\tilde{p}_{2i}^R+\tilde{q}_{2i}^R)\frac{\beta}{1-\gamma})+$$
$$\bar{l}_{ji}^R(\hat{p}_{1i}^I+\hat{q}_{1i}^I)(\Gamma_{2i}^I+(\hat{p}_{2i}^I+\hat{q}_{2i}^I)\frac{\beta}{1-\gamma})+\bar{l}_{ji}^I(\hat{p}_{1i}^R+\hat{q}_{1i}^R)(\Gamma_{2i}^R+(\hat{p}_{2i}^R+\hat{q}_{2i}^R)\frac{\beta}{1-\gamma})],$$

$$\bar{a}_{ij}^R=\sup_{t\in\mathbf{R}}|a_{ij}^R(t)|,\bar{a}_{ij}^I=\sup_{t\in\mathbf{R}}|a_{ij}^I(t)|,\bar{\alpha}_{ij}^R=\sup_{t\in\mathbf{R}}|\alpha_{ij}^R(t)|,\bar{\alpha}_{ij}^I=\sup_{t\in\mathbf{R}}|\alpha_{ij}^I(t)|,\bar{b}_{ji}^R=\sup_{t\in\mathbf{R}}|b_{ji}^R(t)|,$$
$$\bar{b}_{ji}^I=\sup_{t\in\mathbf{R}}|b_{ji}^I(t)|,\bar{l}_{ji}^R=\sup_{t\in\mathbf{R}}|l_{ji}^R(t)|,\bar{l}_{ji}^I=\sup_{t\in\mathbf{R}}|l_{ji}^I(t)|,\bar{I}_i^R=\sup_{t\in\mathbf{R}}|I_i^R(t)|,\bar{I}_i^I=\sup_{t\in\mathbf{R}}|I_i^I(t)|,$$

$$\gamma=\max\{\max_{1\leqslant i\leqslant n}\{\frac{\delta_i^R}{c_i},\frac{\delta_i^I}{c_i}\},\max_{1\leqslant j\leqslant m}\{\frac{\sigma_j^R}{d_j},\frac{\sigma_j^I}{d_j}\}\}<1,\beta=\max\{\max_{1\leqslant i\leqslant n}\{\frac{\bar{I}_i^R}{c_i},\frac{\bar{I}_i^I}{c_i}\},\max_{1\leqslant j\leqslant m}\{\frac{\bar{J}_i^R}{d_j},\frac{\bar{J}_i^I}{d_j}\}\},$$

$$\delta_i^R=\sum_{j=1}^{m}(\bar{a}_{ij}^RM_{2j}^R(p_{1j}^R+q_{1j}^R)+\bar{a}_{ij}^IM_{2j}^I(p_{1j}^I+q_{1j}^I)+\bar{\alpha}_{ij}^RN_{2j}^R(\bar{p}_{1j}^R+\bar{q}_{1j}^R)+\bar{\alpha}_{ij}^IN_{2j}^I(\bar{p}_{1j}^I+\bar{q}\mathrm{I}_{1j})),$$
$$\delta_i^I=\sum_{j=1}^{m}(\bar{a}_{ij}^RM_{2j}^I(p_{1j}^I+q_{1j}^I)+\bar{a}_{ij}^IM_{2j}^R(p_{1j}^R+q_{1j}^R)+\bar{\alpha}_{ij}^RN_{2j}^I(\bar{p}_{1j}^I+\bar{q}_{1j}^I)+\bar{\alpha}_{ij}^IN_{2j}^R(\bar{p}_{1j}^R+\bar{q}_{1j}^R)),$$
$$\sigma_j^R=\sum_{i=1}^{n}(\bar{b}_{ji}^R\Pi_{2i}^R(\tilde{p}_{1i}^R+\tilde{q}_{1i}^R)+\bar{b}_{ji}^I\Pi_{2i}^I(\tilde{p}_{1i}^I+\tilde{q}_{1i}^I)+\bar{l}_{ji}^R\Gamma_{2i}^R(\hat{p}_{1i}^R+\hat{q}_{1i}^R)+\bar{l}_{ji}^I\Gamma_{2i}^I(\tilde{p}_{1i}^I+\tilde{q}_{1i}^I)),$$
$$\sigma_j^I=\sum_{i=1}^{n}(\bar{b}_{ji}^R\Pi_{2i}^I(\tilde{p}_{1i}^I+\tilde{q}_{1i}^I)+\bar{b}_{ji}^I\Pi_{2i}^R(\tilde{p}_{1i}^R+\tilde{q}_{1i}^R)+\bar{l}_{ji}^R\Gamma_{2j}^I(\hat{p}_{1i}^I+\hat{q}_{1i}^I)+\bar{l}_{ji}^I\Gamma_{2i}^R(\hat{p}_{1i}^R+\hat{q}_{1i}^R)),$$
$$\bar{J}_j^R=\sup_{t\in\mathbf{R}}|J_j^R(t)|,\bar{J}_j^I=\sup_{t\in\mathbf{R}}|J_j^I(t)|,j=1,2,\cdots,m,i=1,2,\cdots,n。$$

用 $AP(\mathbf{R},\mathbf{R}^{2(n+m)})$ 表示从 $\mathbf{R}$ 到 $\mathbf{R}^{2(n+m)}$ 的全体概周期函数构成的集合，设

$\Omega=\{\boldsymbol{U}(t)\mid\boldsymbol{U}(t)=(u_1^R(t),u_1^I(t),\cdots,u_n^R(t),u_n^I(t),v_1^R(t),v_1^I(t),\cdots,v_m^R(t),\boldsymbol{v}_m^I(t))^{\mathrm{T}}\in AP(\mathbf{R},\mathbf{R}^{(2(n+m)})\}$。

对任意的 $\boldsymbol{U}(t)\in\Omega$ 定义范数 $\|\boldsymbol{U}\|=\max\{\max\limits_{1\leqslant i\leqslant n}\sup\limits_{t\in\mathbf{R}}\{u_i^R(t),u_i^I(t)\},\max\limits_{1\leqslant j\leqslant m}\sup\limits_{t\in\mathbf{R}}\{v_j^R(t),u_j^I(t)\}\}$，则在该范数下 Ω 成为一个 Banach 空间。

定理 1　若条件 $(C_3)\sim(C_5)$ 成立，则系统(4)在 $\Omega^*=\{\boldsymbol{U}\mid\boldsymbol{U}\in\Omega,\|\boldsymbol{U}\|\leqslant\frac{\beta}{1-\gamma}\}$ 中存在唯一的一致稳定的概周期解。

证明　对于任意的

$$\boldsymbol{\Psi}(t)=(\Psi_1^{\mathrm{R}}(t),\Psi_1^{\mathrm{I}}(t),\cdots,\Psi_n^{\mathrm{R}}(t),\Psi_n^{\mathrm{I}}(t),\Psi_{n+1}^{\mathrm{R}}(t),\Psi_{n+1}^{\mathrm{I}}(t),\cdots,\Psi_{n+m}^{\mathrm{R}}(t),\Psi_{n+m}^{\mathrm{I}}(t))^{\mathrm{T}},$$

考虑线性概周期微分系统

$$\begin{cases}(u_i^{\mathrm{R}})'(t)=-c_i(t)u_i^{\mathrm{R}}(t)+\sum_{j=1}^{m}(a_{ij}^{\mathrm{R}}(t)f_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(t),\Psi_{n+j}^{\mathrm{I}}(t))-a_{ij}^{\mathrm{I}}(t)f_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(t),\Psi_{n+j}^{\mathrm{I}}(t)))+\\ \quad\sum_{j=1}^{m}(\alpha_{ij}^{\mathrm{R}}(t)h_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(t-\tau_{ij}(t)),\Psi_{n+j}^{\mathrm{I}}(t-\tau_{ij}(t)))-\alpha_{ij}^{\mathrm{I}}(t)h_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(t-\tau_{ij}(t)),\Psi_{n+j}^{\mathrm{I}}(t-\tau_{ij}(t))))+I_i^{\mathrm{R}}(t),\\ (u_i^{\mathrm{I}})'(t)=-c_i(t)u_i^{\mathrm{I}}(t)+\sum_{j=1}^{m}(a_{ij}^{\mathrm{R}}(t)f_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(t),\Psi_{n+j}^{\mathrm{I}}(t))+a_{ij}^{\mathrm{I}}(t)f_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(t),\Psi_{n+j}^{\mathrm{I}}(t)))+\\ \quad\sum_{j=1}^{m}(\alpha_{ij}^{\mathrm{R}}(t)h_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(t-\tau_{ij}(t)),\Psi_{n+j}^{\mathrm{I}}(t-\tau_{ij}(t)))+\alpha_{ij}^{\mathrm{I}}(t)h_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(t-\tau_{ij}(t)),\Psi_{n+j}^{\mathrm{I}}(t-\tau_{ij}(t))))+I_i^{\mathrm{I}}(t),\\ (v_j^{\mathrm{R}})'(t)=-d_j(t)v_j^{\mathrm{R}}(t)+\sum_{i=1}^{n}(b_{ji}^{\mathrm{R}}(t)g_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(t),\Psi_i^{\mathrm{I}}(t))-b_{ji}^{\mathrm{I}}(t)g_i^{\mathrm{I}}(\Psi_i^{\mathrm{R}}(t),\Psi_i^{\mathrm{I}}(t)))+\\ \quad\sum_{j=1}^{m}(l_{ji}^{\mathrm{R}}(t)\eta_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(t-\omega_{ji}(t)),\Psi_i^{\mathrm{I}}(t-\omega_{ji}(t)))-l_{ji}^{\mathrm{I}}(t)\eta_i^{\mathrm{I}}(\Psi_i^{\mathrm{R}}(t-\omega_{ji}(t)),\Psi_i^{\mathrm{I}}(t-\omega_{ji}(t))))+J_j^{\mathrm{R}}(t),\\ (v_j^{\mathrm{I}})'(t)=-d_j(t)v_j^{\mathrm{I}}(t)+\sum_{i=1}^{n}(b_{ji}^{\mathrm{R}}(t)g_i^{\mathrm{I}}(\Psi_i^{\mathrm{R}}(t),\Psi_i^{\mathrm{I}}(t))+b_{ji}^{\mathrm{I}}(t)g_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(t),\Psi_i^{\mathrm{I}}(t)))+\\ \quad\sum_{i=1}^{n}(l_{ji}^{\mathrm{R}}(t)\eta_i^{\mathrm{I}}(\Psi_i^{\mathrm{R}}(t-\omega_{ji}(t)),\Psi_i^{\mathrm{I}}(t-\omega_{ji}(t)))+l_{ji}^{\mathrm{I}}(t)\eta_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(t-\omega_{ji}(t)),\Psi_i^{\mathrm{I}}(t-\omega_{ji}(t))))+J_j^{\mathrm{I}}(t)。\end{cases}\tag{8}$$

由条件(C_3)、引理 1、引理 2 可得系统(8)存在唯一的概周期解

$$\boldsymbol{U}_\Psi(t)=((u_\Psi)_1^{\mathrm{R}}(t),(u_\Psi)_1^{\mathrm{I}}(t),\cdots,(u_\Psi)_n^{\mathrm{R}}(t),(u_\Psi)_n^{\mathrm{I}},(v_\Psi)_1^{\mathrm{R}}(t),(v_\Psi)_1^{\mathrm{I}}(t),\cdots,(v_\Psi)_m^{\mathrm{R}}(t),(v_\Psi)_m^{\mathrm{I}}(t))^{\mathrm{T}},$$

其中

$$\begin{cases}(u_\Psi)_i^{\mathrm{R}}(t)=\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t c_i(u)\mathrm{d}u}(\sum_{j=1}^{m}(a_{ij}^{\mathrm{R}}(s)f_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(s),\Psi_{n+j}^{\mathrm{I}}(s))-a_{ij}^{\mathrm{I}}(s)f_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(s),\Psi_{n+j}^{\mathrm{I}}(s)))+\\ \quad\sum_{j=1}^{m}(\alpha_{ij}^{\mathrm{R}}(s)h_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(s-\tau_{ij}(s)),\Psi_{n+j}^{\mathrm{I}}(s-\tau_{ij}(s)))-\alpha_{ij}^{\mathrm{I}}(s)h_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(s-\tau_{ij}(s)),\Psi_{n+j}^{\mathrm{I}}(s-\tau_{ij}(s))))+I_i^{\mathrm{R}}(s))\mathrm{d}s,\\ (u_\Psi)_i^{\mathrm{I}}(t)=\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t c_i(u)\mathrm{d}u}(\sum_{j=1}^{m}(a_{ij}^{\mathrm{R}}(s)f_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(s),\Psi_{n+j}^{\mathrm{I}}(s))+a_{ij}^{\mathrm{I}}(s)f_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(s),\Psi_{n+j}^{\mathrm{I}}(s)))+\\ \quad\sum_{j=1}^{m}(\alpha_{ij}^{\mathrm{R}}(s)h_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(s-\tau_{ij}(s)),\Psi_{n+j}^{\mathrm{I}}(s-\tau_{ij}(s)))+\alpha_{ij}^{\mathrm{I}}(s)h_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(s-\tau_{ij}(s)),\Psi_{n+j}^{\mathrm{I}}(s-\tau_{ij}(s))))+I_i^{\mathrm{I}}(s))\mathrm{d}s,\\ (v_\Psi)_j^{\mathrm{R}}(t)=\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t d_j(u)\mathrm{d}u}(\sum_{i=1}^{n}(b_{ji}^{\mathrm{R}}(s)g_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(s),\Psi_i^{\mathrm{I}}(s))-b_{ji}^{\mathrm{I}}(s)g_i^{\mathrm{I}}(s)(\Psi_i^{\mathrm{R}}(s),\Psi_i^{\mathrm{I}}(s)))+\\ \quad\sum_{i=1}^{n}(l_{ji}^{\mathrm{R}}(s)\eta_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(s-\omega_{ji}(s)),\Psi_i^{\mathrm{I}}(s-\omega_{ji}(s)))-l_{ji}^{\mathrm{I}}(s)\eta_i^{\mathrm{I}}(\Psi_i^{\mathrm{R}}(s-\omega_{ji}(s)),\Psi_i^{\mathrm{I}}(s-\omega_{ji}(s))))+J_j^{\mathrm{R}}(s))\mathrm{d}s,\\ (v_\Psi)_j^{\mathrm{I}}(t)=\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t d_j(u)\mathrm{d}u}(\sum_{i=1}^{n}b_{ji}^{\mathrm{R}}(s)g_i^{\mathrm{I}}(\Psi_i^{\mathrm{R}}(s),\Psi_i^{\mathrm{I}}(s))+b_{ji}^{\mathrm{I}}(s)g_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(s),\Psi_i^{\mathrm{I}}(s)))+\\ \quad\sum_{i=1}^{n}(l_{ji}^{\mathrm{R}}(s)\eta_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(s-\omega_{ji}(s)),\Psi_i^{\mathrm{I}}(s-\omega_{ji}(s)))+l_{ji}^{\mathrm{I}}(s)\eta_i^{\mathrm{R}}(\Psi_i^{\mathrm{R}}(s-\omega_{ji}(s)),\Psi_i^{\mathrm{I}}(s-\omega_{ji}(s))))+J_j^{\mathrm{I}}(s))\mathrm{d}s。\end{cases}\tag{9}$$

因为 $\boldsymbol{U}_\Psi(t)\in\Omega$,所以可以在 Ω^* 上定义映射 $\boldsymbol{\Psi}:\Omega^*\to\Omega$ 为 $\boldsymbol{\Psi}_\Psi=\boldsymbol{U}_\Psi$。接下来证明 $\boldsymbol{U}_\Psi(t)\in\Omega^*$。由(9)式及条件($\mathrm{C}_4$)($\mathrm{C}_5$)可得

$$|(u_\Psi)_i^{\mathrm{R}}(t)|=|\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t c_i(u)\mathrm{d}u}(\sum_{j=1}^{m}(a_{ij}^{\mathrm{R}}(s)f_j^{\mathrm{R}}(\Psi_{n+j}^{\mathrm{R}}(s),\Psi_{n+j}^{\mathrm{I}}(s))-a_{ij}^{\mathrm{I}}(s)f_j^{\mathrm{I}}(\Psi_{n+j}^{\mathrm{R}}(s),\Psi_{n+j}^{\mathrm{I}}(s)))+$$

$$\sum_{j=1}^{m}(\alpha_{ij}^{R}(s)h_j^{R}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))-\alpha_{ij}^{I}(s)h_j^{I}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s))))+$$

$$I_i^{R}(s))\mathrm{d}s\,|\leqslant$$

$$\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t c_i(u)\mathrm{d}u}(\sum_{j=1}^{m}(\overline{a}_{ij}^{R}\,|\,f_{1j}^{R}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{R}(s))\,\|\,f_{2j}^{R}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{R}(s))\,|+$$

$$\overline{a}_{ij}^{I}\,|\,f_{1j}^{I}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{R}(s))\,\|\,f_{2j}^{I}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{R}(s))\,|)+$$

$$\sum_{j=1}^{m}(\overline{\alpha}_{ij}^{R}\,|\,h_{1j}^{R}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))\,\|\,h_{2j}^{R}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))\,|+$$

$$\overline{\alpha}_{ij}^{I}\,|\,h_{1j}^{I}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))\,\|\,h_{2j}^{I}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))\,|)+\overline{I}_i^{R})\mathrm{d}s\leqslant$$

$$\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t c_i(u)\mathrm{d}u}(\sum_{j=1}^{m}(\overline{a}_{ij}^{R}M_{2j}^{R}(p_{1j}^{R}+q_{1j}^{R})+\overline{a}_{ij}^{I}M_{2j}^{I}(p_{1j}^{I}+q_{1j}^{I}))\,\|\,\Psi\,\|\,+$$

$$\sum_{j=1}^{m}(\overline{\alpha}_{ij}^{R}N_{2j}^{R}(\overline{p}_{1j}^{R}+\overline{q}_{1j}^{R})+\overline{\alpha}_{ij}^{I}(\overline{p}_{1j}^{I}+\overline{q}_{1j}^{I})\,\|\,\Psi\,\|\,+\overline{I}_i^{R})\mathrm{d}s\leqslant$$

$$\frac{1}{c_1}((\sum_{j=1}^{m}(\overline{a}_{ij}^{R}M_{2j}^{R}(p_{1j}^{R}+q_{1j}^{R})+\overline{a}_{ij}^{I}M_{2j}^{I}(p_{1j}^{I}+q_{1j}^{I}))+\sum_{j=1}^{m}(\overline{\alpha}_{ij}^{R}N_{2j}^{R}(\overline{p}_{1j}^{R}+\overline{q}_{1j}^{R})+$$

$$\overline{\alpha}_{ij}^{I}N_{2j}^{I}(\overline{p}_{1j}^{I}+\overline{q}_{1j}^{I}))\,\|\,\Psi\,\|\,+\overline{I}_i^{R})=\frac{\delta_i^{R}}{c_i}+\frac{\overline{I}_i^{R}}{c_i}\leqslant\gamma\frac{\beta}{1-\gamma}+\beta=\frac{\beta}{1-\gamma},$$

类似于上面的计算，可得

$$|\,(u_\Psi)_i^{I}(t)\,|\leqslant\frac{1}{c_i}((\sum_{j=1}^{m}\overline{a}_{ij}^{R}M_{2j}^{I}(p_{1j}^{I}+q_{1j}^{I})+\overline{a}_{ij}^{I}M_{2j}^{R}(p_{1j}^{R}+q_{1j}^{R})+\overline{\alpha}_{ij}^{R}N_{2j}^{I}(\overline{p}_{1j}^{I}+\overline{q}_{1j}^{I})+$$

$$\overline{\alpha}_{ij}^{I}N_{2j}^{R}(\overline{p}_{1j}^{R}+\overline{q}_{1j}^{R}))\,\|\,\Psi\,\|\,+\overline{I}_i^{I})=\frac{\delta_i^{I}}{c_i}+\frac{\overline{I}_i^{I}}{c_i}\leqslant\gamma\frac{\beta}{1-\gamma}+\beta=\frac{\beta}{1-\gamma},$$

$$|\,(v_\Psi)_j^{R}(t)\,|\leqslant\frac{1}{d_j}((\sum_{i=1}^{n}\overline{b}_{ji}^{R}\Pi_{2i}^{R}(\tilde{p}_{1i}^{R}+\tilde{q}_{1i}^{R})+\overline{b}_{ji}^{I}\Pi_{2i}^{I}(\tilde{p}_{1i}^{I}+\tilde{q}_{1i}^{I})+\overline{l}_{ji}^{R}\Gamma_{2i}^{R}(\hat{p}_{1i}^{R}+\hat{q}_{1i}^{R})+$$

$$\overline{l}_{ji}^{I}\Gamma_{2i}^{I}(\hat{p}_{1i}^{I}+\hat{q}_{1i}^{I}))\,\|\,\Psi\,\|\,+\overline{J}_j^{R})=\frac{\sigma_j^{R}}{d_j}+\frac{\overline{J}_j^{R}}{d_j}\leqslant\gamma\frac{\beta}{1-\gamma}+\beta=\frac{\beta}{1-\gamma},$$

$$|\,(v_\Psi)_j^{I}(t)\,|\leqslant\frac{1}{d_j}((\sum_{i=1}^{n}\overline{b}_{ji}^{R}\Pi_{2i}^{I}(\tilde{p}_{1i}^{I}+\tilde{q}_{1i}^{I})+\overline{b}_{ji}^{I}\Pi_{2i}^{R}(\tilde{p}_{1i}^{R}+\tilde{q}_{1i}^{R})+\overline{l}_{ji}^{R}\Gamma_{2i}^{I}(\hat{p}_{1i}^{I}+\hat{q}_{1i}^{I})+$$

$$\overline{l}_{ji}^{I}\Gamma_{2i}^{R}(\hat{p}_{1i}^{R}+\hat{q}_{1i}^{R}))\,\|\,\Psi\,\|\,+\overline{J}_j^{I})=\frac{\sigma_j^{I}}{d_j}+\frac{\overline{J}_j^{I}}{d_j}\leqslant\gamma\frac{\beta}{1-\gamma}+\beta=\frac{\beta}{1-\gamma}。$$

综上所述

$$\|\,\boldsymbol{U}_\Psi\,\|\,=\max\{\max_{1\leqslant i\leqslant n}\sup_{t\in\mathbf{R}}|\,(u_\Psi)_i^{R}(t)\,|,\max_{1\leqslant i\leqslant n}\sup_{t\in\mathbf{R}}|\,(u_\Psi)_i^{I}(t)\,|,\max_{1\leqslant j\leqslant m}\sup_{t\in\mathbf{R}}|\,(v_\Psi)_j^{R}(t),$$

$$\max_{1\leqslant j\leqslant m}\sup_{t\in\mathbf{R}}|\,(v_\Psi)_j^{I}(t)\,|\}\leqslant\frac{\beta}{1-r},$$

即 $\boldsymbol{U}_\Psi(t)\in\Omega^*(t)$，从而对任意的 $\boldsymbol{\psi}\in\Omega^*$ 都有 $\boldsymbol{\Psi}_\Psi\in\Omega^*$，映射 $\boldsymbol{\Psi}$ 是从 Ω^* 到 Ω^* 的自反映射。

下面证明 $\boldsymbol{\Psi}:\Omega^*\to\Omega^*$ 是一个压缩映射。对任意的 $\boldsymbol{\psi},\boldsymbol{\rho}\in\Omega^*$ 设

$$\boldsymbol{\psi}(t)=(\Psi_1^{R}(t),\Psi_1^{I}(t),\cdots,\Psi_n^{R}(t),\Psi_n^{I}(t),\Psi_{n+1}^{R}(t),\Psi_{n+1}^{I}(t),\cdots,\Psi_{n+m}^{R}(t),\Psi_{n+m}^{I}(t))^{T},$$

$$\boldsymbol{\rho}(t)=(\rho_1^{R}(t),\rho_1^{I}(t),\cdots,\rho_n^{R}(t),\rho_n^{I}(t),\rho_{n+1}^{R}(t),\rho_{n+1}^{I}(t),\cdots,\rho_{n+m}^{R}(t),\rho_{n+m}^{I}(t))^{T},$$

则

$$|\,(\boldsymbol{\Psi}_\Psi)_i^{R}(t)-(\boldsymbol{\Psi}_\rho)_i^{R}(t)\,|=|\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t c_i(u)\mathrm{d}u}(\sum_{j=1}^{m}(a_{ij}^{R}(s)(f_j^{R}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{I}(s))-$$

$$f_j^{R}(\rho_{n+j}^{R}(s),\rho_{n+j}^{I}(s)))-a_{ij}^{I}(s)(f_j^{I}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{I}(s))-f_j^{I}(\rho_{n+j}^{R}(s),\rho_{n+j}^{I}(s))))+$$

$$\sum_{j=1}^{m}(\alpha_{ij}^{R}(s)(h_{j}^{R}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))-h_{j}^{R}(\rho_{n+j}^{R}(s-\tau_{ij}(s)),\rho_{n+j}^{I}(s-\tau_{ij}(s))))-$$

$$\alpha_{ij}^{I}(s)(h_{j}^{I}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))-h_{j}^{I}(\rho_{n+j}^{R}(s-\tau_{ij}(s)),\rho_{n+j}^{I}(s-\tau_{ij}(s))))))\mathrm{d}s\,|\leqslant$$

$$\int_{-\infty}^{t}\mathrm{e}^{-\int_{s}^{t}c_i(u)\mathrm{d}u}(\sum_{j=1}^{m}(|a_{ij}^{R}(s)\|f_{1j}^{R}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{I}(s))f_{2j}^{R}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{I}(s))-$$

$$f_{1j}^{R}(\rho_{n+j}^{R}(s),\rho_{n+j}^{I}(s))f_{2j}^{R}(\rho_{n+j}^{R}(s),\rho_{n+j}^{I}(s))|+$$

$$|a_{ij}^{I}(s)\|f_{1j}^{I}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{I}(s))f_{2j}^{I}(\Psi_{n+j}^{R}(s),\Psi_{n+j}^{I}(s))-f_{1j}^{I}(\rho_{n+j}^{R}(s),\rho_{n+j}^{I}(s))f_{2j}^{I}(\rho_{n+j}^{R}(s),\rho_{n+j}^{I}(s))|)+$$

$$\sum_{j=1}^{m}(|\alpha_{ij}^{R}(s)\|h_{1j}^{R}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))h_{2j}^{R}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))-$$

$$h_{1j}^{R}(\rho_{n+j}^{R}(s-\tau_{ij}(s)),\rho_{n+j}^{I}(s-\tau_{ij}(s)))h_{2j}^{R}(\rho_{n+j}^{R}(s-\tau_{ij}(s)),\rho_{n+j}^{I}(s-\tau_{ij}(s)))|+$$

$$|\alpha_{ij}^{I}(s)\|h_{1j}^{I}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))h_{2j}^{I}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))-$$

$$h_{1j}^{I}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))h_{2j}^{I}(\Psi_{n+j}^{R}(s-\tau_{ij}(s)),\Psi_{n+j}^{I}(s-\tau_{ij}(s)))|))\mathrm{d}s\leqslant$$

$$\int_{-\infty}^{t}\mathrm{e}^{-\int_{s}^{t}c_i(u)\mathrm{d}u}\{\sum_{j=1}^{m}\bar{a}_{ij}^{R}(M_{2j}^{R}(p_{1j}^{R}+q_{1j}^{R})+(p_{1j}^{R}+q_{1j}^{R})\frac{\beta}{1-\gamma}(p_{2j}^{R}+q_{2j}^{R}))\|\Psi-\rho\|+$$

$$\bar{a}_{ij}^{I}(M_{2j}^{I}(p_{1j}^{I}+q_{1j}^{I})+(p_{1j}^{I}+q_{1j}^{I})\frac{\beta}{1-\gamma}(p_{2j}^{I}+q_{2j}^{I}))\|\Psi-\rho\|+$$

$$\sum_{j=1}^{m}(\bar{\alpha}_{ij}^{R}(N_{2j}^{R}(\bar{p}_{1j}^{R}+\bar{q}_{1j}^{R})+(\bar{p}_{1j}^{R}+\bar{q}_{1j}^{R})\frac{\beta}{1-\gamma}(\bar{p}_{2j}^{R}+\bar{q}_{2j}^{R}))))\}\mathrm{d}s\,\|\Psi-\rho\|。$$

类似于上面的计算,还可以得到

$$|(\Psi_{\Psi})_i^{I}(t)-(\Psi_{\rho})_i^{I}(t)|\leqslant$$

$$\frac{1}{c_i}\{\sum_{j=1}^{m}[\bar{a}_{ij}^{R}(p_{1j}^{I}+q_{1j}^{I})(M_{2j}^{I}+(p_{2j}^{I}+q_{2j}^{I})\frac{\beta}{1-\gamma})+\bar{a}_{ij}^{I}(p_{1j}^{R}+q_{1j}^{R})(M_{2j}^{R}+(p_{2j}^{R}+q_{2j}^{R})\frac{\beta}{1-\gamma})+$$

$$\bar{\alpha}_{ij}^{R}(\bar{p}_{1j}^{I}+\bar{q}_{1j}^{I})(N_{2j}^{I}+(\bar{p}_{2j}^{I}+\bar{q}_{2j}^{I})\frac{\beta}{1-\gamma})+$$

$$\bar{\alpha}_{ij}^{I}(\bar{p}_{1j}^{R}+\bar{q}_{1j}^{R})(N_{2j}^{R}+(\bar{p}_{2j}^{R}+\bar{q}_{2j}^{R})\frac{\beta}{1-\gamma})]\}\|\Psi-\rho\|=\frac{\mu_i^{I}}{c_i}\|\Psi-\rho\|,$$

$$|(\Psi_{\Psi})_{n+j}^{R}(t)-(\Psi_{\rho})_{n+j}^{R}(t)|\leqslant$$

$$\frac{1}{d_j}\{\sum_{i=1}^{n}[\bar{b}_{ji}^{R}(\tilde{p}_{1i}^{R}+\tilde{q}_{1i}^{R})(\Pi_{2i}^{R}+(\tilde{p}_{2i}^{R}+\tilde{q}_{2i}^{R})\frac{\beta}{1-\gamma})+\bar{b}_{ji}^{I}(\tilde{p}_{1i}^{I}+\tilde{q}_{1i}^{I})(\Pi_{2i}^{I}+(\tilde{p}_{2i}^{I}+\tilde{q}_{2i}^{I})\frac{\beta}{1-\gamma})+$$

$$\bar{l}_{ji}^{R}(\hat{p}_{1i}^{R}+\hat{q}_{1i}^{R})(\Gamma_{2i}^{R}+(\hat{p}_{2i}^{R}+\hat{q}_{2i}^{R})\frac{\beta}{1-\gamma})+$$

$$\bar{l}_{ji}^{I}(\hat{p}_{1i}^{I}+\hat{q}_{1i}^{I})(\Gamma_{2i}^{I}+(\hat{p}_{2i}^{I}+\hat{q}_{2i}^{I})\frac{\beta}{1-\gamma})]\}\|\Psi-\rho\|=\frac{\vartheta_j^{R}}{d_j}\|\Psi-\rho\|,$$

$$|(\Psi_{\Psi})_{n+j}^{I}(t)-(\Psi_{\rho})_{n+j}^{I}(t)|\leqslant$$

$$\frac{1}{d_j}\{\sum_{i=1}^{m}[\bar{b}_{ji}^{R}(\tilde{p}_{1i}^{I}+\tilde{q}_{1i}^{I})(\Pi_{2i}^{I}+(\tilde{p}_{2i}^{I}+\tilde{q}_{2i}^{I})\frac{\beta}{1-\gamma})+\bar{b}_{ji}^{I}(\tilde{p}_{1i}^{R}+\tilde{q}_{1i}^{R})(\Pi_{2i}^{R}+(\tilde{p}_{2i}^{R}+\tilde{q}_{2i}^{R})\frac{\beta}{1-\gamma})+$$

$$\bar{l}_{ji}^{R}(\hat{p}_{1i}^{I}+\hat{q}_{1i}^{I})(\Gamma_{2i}^{I}+(\hat{p}_{2i}^{I}+\hat{q}_{2i}^{I})\frac{\beta}{1-\gamma})+$$

$$\bar{l}_{ji}^{I}(\hat{p}_{1i}^{R}+\hat{q}_{1i}^{R})(\Gamma_{2i}^{R}+(\hat{p}_{2i}^{R}+\hat{q}_{2i}^{R})\frac{\beta}{1-\gamma})]\}\|\Psi-\rho\|=\frac{\vartheta_j^{I}}{d_j}\|\Psi-\rho\|。$$

所以

$$\|\Psi_{\Psi}-\Psi_{\rho}\|=\max\{\max_{1\leqslant i\leqslant n}\sup_{t\in\mathbf{R}}\{\|(\Psi_{\Psi})_i^{R}(t)-(\Psi_{\rho})_i^{R}(t)\|,\|(\Psi_{\Psi})_i^{I}(t)-(\Psi_{\rho})_i^{I}(t)\|\},$$

$$\max_{1\leqslant j\leqslant m}\sup_{t\in\mathbf{R}}\{\|(\Psi_{\Psi})_{n+j}^{R}(t)-(\Psi_{\rho})_{n+j}^{R}(t)\|,\|(\Psi_{\Psi})_{n+j}^{I}(t)-(\Psi_{\rho})_{n+j}^{I}(t)\|\}\}\leqslant\lambda\|\Psi-\rho\|。$$

因为$\lambda<1$,所以映射$\Psi:\Omega^{*}\rightarrow\Omega^{*}$是一个压缩映射。由Banach空间不动点定理可得,Ψ在Ω^{*}中存在唯一

的不动点，因此系统(2) 在 Ω^* 中存在唯一的概周期解

$$U(t)=(u_1^R(t),u_1^I(t),\cdots,u_n^R(t),u_n^I(t),v_1^R(t),v_1^I(t),\cdots,v_m^R(t),v_m^I(t))^T,$$

且 $\|U\|\leqslant\frac{\beta}{1-\gamma}$。最后证明系统(2) 的概周期解 $U(t)$ 是一致稳定的。设 $U(t)$ 是系统(2) 具有初值 $\boldsymbol{\Phi}$ 的解，$X(t)=(x_1^R(t),x_1^I(t),\cdots,x_n^R(t),x_n^I(t),y_1^R(t),\cdots,y_m^R(t),y_m^I(t))^T$ 是系统(2) 具有初值 $\boldsymbol{\Lambda}$ 的任一解，则

$$\begin{aligned}
u_i^R(t)&=\varphi_i^R(t_0)e^{-\int_{t_0}^t c_i(u)du}+\int_{t_0}^t e^{-\int_s^t c_i(u)du}\Big[\sum_{j=1}^m(a_{ij}^R(s)f_j^R(v_j^R(s),v_j^I(s))-a_{ij}^I(s)f_j^I(v_j^R(s),v_j^I(s)))+\\
&\sum_{j=1}^m(\alpha_{ij}^R(s)h_j^R(v_j^R(s-\tau_{ij}(s)),v_j^I(s-\tau_{ij}(s)))-\alpha_{ij}^I(s)h_j^I(v_j^R(s-\tau_{ij}(s)),v_j^I(s-\tau_{ij}(s))))+I_i^R(s)\Big]ds,\\
u_i^I(t)&=\varphi_i^I(t_0)e^{-\int_{t_0}^t c_i(u)du}+\int_{t_0}^t e^{-\int_s^t c_i(u)du}\Big[\sum_{j=1}^m(a_{ij}^R(s)f_j^I(v_j^R(s),v_j^I(s))+a_{ij}^I(s)f_j^R(v_j^R(s),v_j^I(s)))+\\
&\sum_{j=1}^m(\alpha_{ij}^R(s)h_j^I(v_j^R(s-\tau_{ij}(s)),v_j^I(s-\tau_{ij}(s)))+\alpha_{ij}^I(s)h_j^R(v_j^R(s-\tau_{ij}(s)),v_j^I(s-\tau_{ij}(s))))+I_i^I(s)\Big]ds,\\
v_j^R(t)&=\varphi_{n+j}^R(t_0)e^{-\int_{t_0}^t d_j(u)du}+\int_{t_0}^t e^{-\int_s^t d_j(u)du}\Big[\sum_{i=1}^n(b_{ji}^R(s)g_i^R(u_i^R(s),u_i^I(s))-b_{ji}^I(s)g_i^I(u_i^R(s),u_i^I(s)))+\\
&\sum_{i=1}^n(l_{ji}^R(s)\eta_i^R(u_i^R(s-\omega_{ji}(s)),u_i^I(s-\tau_{ij}(s)))-l_{ji}^I(s)\eta_i^I(u_i^R(s-\omega_{ji}(s)),u_i^I(s-\omega_{ji}(s))))+J_j^R(s)\Big]ds,\\
v_j^I(t)&=\varphi_{n+j}^I(t_0)e^{-\int_{t_0}^t d_j(u)du}+\int_{t_0}^t e^{-\int_s^t d_j(u)du}\Big[\sum_{i=1}^n(b_{ji}^R(s)g_i^I(u_i^R(s),u_i^I(s))+b_{ji}^I(s)g_i^R(u_i^R(s),u_i^I(s)))+\\
&\sum_{i=1}^n(l_{ji}^R(s)\eta_i^I(u_i^R(s-\omega_{ji}(s)),u_i^I(s-\omega_{ji}(s)))+l_{ji}^I(s)\eta_i^R(u_i^R(s-\omega_{ji}(s)),u_i^I(s-\omega_{ji}(s))))+J_j^I(s)\Big]ds。
\end{aligned}\tag{10}$$

$$\begin{aligned}
x_i^R(t)&=\Lambda_i^R(t_0)e^{-\int_{t_0}^t c_i(u)du}+\int_{t_0}^t e^{-\int_s^t c_i(u)du}\Big[\sum_{j=1}^m(a_{ij}^R(s)f_j^R(y_j^R(s),y_j^I(s)),-a_{ij}^I(s)f_j^I(y_j^R(s),y_j^I(s)))+\\
&\sum_{j=1}^m(\alpha_{ij}^R(s)h_j^R(y_j^R(s-\tau_{ij}(s)),y_j^I(s-\tau_{ij}(s)))-\alpha_{ij}^I(s)h_j^I(y_j^R(s-\tau_{ij}(s)),y_j^I(s-\tau_{ij}(s))))+I_i^R(s)\Big]ds,\\
x_i^I(t)&=\Lambda_i^I(t_0)e^{-\int_{t_0}^t c_i(u)du}+\int_{t_0}^t e^{-\int_s^t c_i(u)du}\Big[\sum_{j=1}^m(a_{ij}^R(s)f_j^I(y_j^R(s)),y_j^I(s))+a_{ij}^I(s)f_j^R(y_j^R(s),y_j^I(s)))+\\
&\sum_{j=1}^m(\alpha_{ij}^R(s)h_j^I(y_j^R(s-\tau_{ij}(s)),y_j^I(s-\tau_{ij}(s)))+\alpha_{ij}^I(s)h_j^R(y_j^R(s-\tau_{ij}(s)),y_j^I(s-\tau_{ij}(s))))+I_i^I(s)\Big]ds,\\
y_j^R(t)&=\Lambda_{n+j}^R(t_0)e^{-\int_{t_0}^t d_j(u)du}+\int_{t_0}^t e^{-\int_s^t d_j(u)du}\Big[\sum_{i=1}^n(b_{ji}^R(s)g_i^R(x_i^R(s),x_i^I(s))-b_{ji}^I(s)g_i^I(x_i^R(s),x_i^I(s)))+\\
&\sum_{i=1}^n(l_{ji}^R(s)\eta_i^R(x_i^R(s-\omega_{ji}(s)),x_i^I(s-\tau_{ij}(s)))-l_{ji}^I(s)\eta_i^I(x_i^R(s-\omega_{ji}(s)),x_i^I(s-\omega_{ji}(s))))+J_j^R(s)\Big]ds,\\
y_j^I(t)&=\Lambda_{n+j}^I(t_0)e^{-\int_{t_0}^t d_j(u)du}+\int_{t_0}^t e^{-\int_s^t d_j(u)du}\Big[\sum_{i=1}^n(b_{ji}^R(s)g_i^I(x_i^R(s),x_i^I(s))+b_{ji}^I(s)g_i^R(x_i^R(s),x_i^I(s)))+\\
&\sum_{i=1}^n(l_{ji}^R(s)\eta_i^I(x_i^R(s-\omega_{ji}(s)),x_i^I(s-\omega_{ji}(s)))+l_{ji}^I(s)\eta_i^R(x_i^R(s-\omega_{ji}(s)),x_i^I(s-\omega_{ji}(s))))+J_j^I(s)\Big]ds。
\end{aligned}\tag{11}$$

对任意的 $\varepsilon>0$，取 $\delta=\frac{\varepsilon}{2}(1-\lambda)>0$，则当 $\|\boldsymbol{\Phi}-\boldsymbol{\Lambda}\|<\delta$ 时，必有

$$\|U(t)-X(t)\|<\varepsilon,t\geqslant t_0。\tag{12}$$

若不然，则一定存在 $t_1>t_0$ 使得

$$\|U(t)-X(t)\|<\varepsilon,t_0<t<t_1,\tag{13}$$

且

$$\|U(t_1)-X(t_1)\|=\varepsilon。\tag{14}$$

此时,由式(10)(11)(13)(14)可得

$$|u_i^R(t_1)-x_i^R(t_1)|=$$

$$|\varphi_i^R(t_0)-\Lambda_i^R(t_0)|e^{-\int_{t_0}^{t_1}c_i(u)du}+\int_{t_0}^{t_1}e^{-\int_s^{t_1}c_i(u)du}[\sum_{j=1}^{m}(a_{ij}^R(s)(f_j^R(v_j^R(s),v_j^I(s))-f_j^R(y_j^R(s),y_j^I(s)))-$$

$$a_{ij}^I(s)(f_j^I(v_j^R(s),v_j^I(s))-f_j^I(y_j^R(s),y_j^I(s))))+$$

$$\sum_{j=1}^{m}(\alpha_{ij}^R(h_j^R(v_j^R(s-\tau_{ij}(s)),v_j^I(s-\tau_{ij}(s)))-h_j^R(y_j^R(s-\tau_{ij}(s)),y_j^I(s-\tau_{ij}(s))))-$$

$$\alpha_{ij}^I(s)(h_j^I(v_j^R(s-\tau_{ij}(s)),v_j^I(s-\tau_{ij}(s)))-h_j^R(y_j^I(s-\tau_{ij}(s)),y_j^I(s-\tau_{ij}(s)))))]ds<$$

$$\delta e^{-\int_{t_0}^{t_1}c_i(u)du}+\int_{t_0}^{t_1}\varepsilon e^{-\int_s^{t_1}c_i(u)du}\{\sum_{j=1}^{m}[\overline{a_{ij}^R}(p_{1j}^R+q_{1j}^R)(M_{2j}^R+(p_{2j}^R+q_{2j}^R)\frac{\beta}{1-\gamma})+$$

$$\overline{a_{ij}^I}(p_{1j}^I+q_{1j}^I)(M_{2j}^I+(p_{2j}^I+q_{2j}^I)\frac{\beta}{1-\gamma})+\overline{\alpha_{ij}^R}(\overline{p}_{1j}^R+\overline{q}_{1j}^R)(N_{2j}^R+(\overline{p}_{2j}^R+\overline{q}_{2j}^R)\frac{\beta}{1-\gamma})+$$

$$\overline{\alpha_{ij}^I}(\overline{p}_{1j}^I+\overline{q}_{1j}^I)(N_{2j}^I+(\overline{p}_{2j}^I+\overline{q}_{2j}^I)\frac{\beta}{1-\gamma})]\}<\delta+\varepsilon\frac{\mu_i^R}{c_i},$$

同理可得

$$|u_i^I(t_1)-x_i^I(t_1)|<\delta+\varepsilon\frac{\mu_i^I}{c_i},\ |v_j^R(t_1)-y_j^R(t_1)|<\delta+\varepsilon\frac{\vartheta_j^R}{d_j},\ |v_j^I(t_1)-y_j^I(t_1)|<\delta+\varepsilon\frac{\vartheta_j^I}{d_j},$$

所以

$$\varepsilon=\|\boldsymbol{U}(t_1)-\boldsymbol{X}(t_1)\|=$$

$$\max\{\max_{1\leqslant i\leqslant n}\{|u_i^R(t_1)-x_i^R(t_1)|,|u_i^I(t_1)-x_i^I(t_1)|\},\max_{1\leqslant j\leqslant m}\{|v_j^R(t_1)-y_j^R(t_1)|,|v_j^I(t_1)-y_j^I(t_1)|\}<$$

$$\delta+\varepsilon\lambda=\frac{\varepsilon}{2}(1-\lambda)+\varepsilon\lambda=\frac{\varepsilon(1+\lambda)}{2}<\varepsilon,$$

这与式(14)矛盾,故式(12)成立,系统(4)的概周期解是一致稳定的。

3 数值模拟

考虑如下具有时变时滞的复值 BAM 神经网络

$$\begin{cases}u_i'(t)=-c_i(t)u_i(t)+\sum_{j=1}^{m}a_{ij}(t)f_j(v_j(t))+\sum_{j=1}^{m}\alpha_{ij}(t)h_j(v_j(t-\tau_{ij}(t)))+I_i(t),i=1,2,\\ v_j'(t)=-d_j(t)v_j(t)+\sum_{i=1}^{n}b_{ji}(t)g_i(u_i(t))+\sum_{i=1}^{n}l_{ji}(t)\eta_i(u_i(t-\omega_{ji}(t)))+J_j(t),j=1,2。\end{cases}\tag{15}$$

其中:

$$c_1(t)=6+|\sin(\sqrt{3}t)|,c_2(t)=7-|\cos(\sqrt{2}t)|,d_1(t)=7+\sin^2t,d_2(t)=8-\cos^2t,$$

$$a_{11}(t)=0.1\sin(\sqrt{2}t)+i0.2\cos(3t),a_{12}(t)=0.2\cos(2t)-i0.1\sin(\sqrt{3}t),a_{21}(t)=0.3\sin^2t+i0.5|\cos t|,$$

$$a_{22}(t)=0.5\cos^3t-i0.3|\sin(\sqrt{2}t)|,\alpha_{11}(t)=0.2|\sin t|+i0.4\cos^2t,\alpha_{12}(t)=0.4|\sin(\sqrt{3}t)|-i0.2\sin t,$$

$$\alpha_{21}(t)=0.1|\sin(\sqrt{2}t)|-i0.3\sin^4t,\alpha_{22}(t)=0.3|\cos(\sqrt{3}t)|+i0.1\cos^2t,b_{11}(t)=0.6\sin^2t+i0.3\cos^2t,$$

$$b_{12}(t)=0.3\cos^4t-i0.6\sin^4t,b_{21}(t)=0.2|\sin(\sqrt{5}t)|+i0.5|\cos t|,b_{22}(t)=0.5|\cos(\sqrt{3}t)|-i0.2\sin^4t,$$

$$l_{11}(t)=0.2\sin^6t+i0.3|\cos(\sqrt{2}t)|,l_{12}(t)=0.3|\sin(\sqrt{2}t)|-i0.2\cos^6t,l_{21}(t)=0.1|\sin t|+i0.3\cos^6t,$$

$$l_{22}(t)=0.2\cos^6t+i0.5\sin^4t,I_1(t)=2\sin^2t-i3\cos(\sqrt{2}t),I_2(t)=3\sin(\sqrt{3}t)+i2\cos^4t,$$

$$J_1(t)=\sin(\sqrt{5}t)+i2\cos(\sqrt{3}t),J_2(t)=2\sin(\sqrt{3}t)-i\cos(\sqrt{5}t),\tau_{11}(t)=\frac{|\sin(\sqrt{2}t)|}{10},\tau_{12}(t)=\frac{\cos^2t}{5},$$

$$\tau_{21}(t)=\frac{\sin^2t}{15},\tau_{22}=\frac{|\cos(\sqrt{3}t)|}{20},\omega_{11}(t)=\frac{\sin^4t}{8},\omega_{12}(t)=\frac{|\sin(\sqrt{5}t)|}{4},\omega_{21}(t)=\omega_{22}(t)=\frac{\cos^2t}{8},$$

$$f_j(v_j)=\frac{(v_j^{R}+v_j^{I})\sin(v_j^{R}+v_j^{I})}{64}+\mathrm{i}\,\frac{v_j^{I}}{64},f_{1j}^{R}(v_j^{R},v_j^{I})=\frac{v_j^{R}+v_j^{I}}{64},f_{2j}^{R}(v_j^{R},v_j^{I})=\sin(v_j^{R}+v_j^{I}),f_{1j}^{I}=\frac{v_j^{I}}{64},f_{2j}^{I}=1,$$

$$g_i(u_i)=\frac{(u_i^{R}+u_i^{I})\sin(u_i^{R}+u_i^{I})}{32}+\mathrm{i}\,\frac{u_i^{I}}{32},g_{1i}^{R}(u_i^{R},u_i^{I})=\frac{u_i^{R}+u_i^{I}}{32},g_{2i}^{R}(u_i^{R},u_i^{I})=\sin(u_i^{R}+u_i^{I}),g_{1i}^{I}=\frac{u_i^{I}}{32},g_{2i}^{I}=1,$$

$$h_j(v_j)=\frac{v_j^{R}\sin v_j^{R}+\mathrm{i}(v_j^{R}+v_j^{I})\cos(v_j^{R}+v_j^{I})}{48},h_{1j}^{R}(v_j^{R},v_j^{I})=\frac{v_j^{R}}{48},h_{2j}^{R}(v_j^{R},v_j^{I})=\sin(v_j^{R}),h_{1j}^{I}(v_j^{R},v_j^{I})=\frac{v_j^{R}+v_j^{I}}{48},$$

$$h_{2j}^{I}(v_j^{R},v_j^{I})=\cos(v_j^{R}+v_j^{I}),\eta_i(u_i)=\frac{u_i^{R}\cos u_i^{R}+\mathrm{i}(u_i^{R}+u_i^{I})\sin(u_i^{R}+u_i^{I})}{96},\eta_{1i}^{R}(u_i^{R},u_i^{I})=\frac{u_i^{R}}{96},$$

$$\eta_{2i}^{R}(u_i^{R},u_i^{I})=\cos u_i^{R},\eta_{1i}^{I}(u_i^{R},u_i^{I})=\frac{u_i^{R}+u_i^{I}}{96},\eta_{2i}^{I}(u_i^{R},u_i^{I})=\sin(u_i^{R}+u_i^{I})。$$

直接计算后，可得

$$c_1=c_2=6,d_1=d_2=7,M_{2j}^{R}=M_{2j}^{I}=N_{2j}^{R}=N_{2j}^{I}=\Pi_{2i}^{R}=\Pi_{2i}^{I}=\Gamma_{2i}^{R}=1,p_{1j}^{R}=q_{1j}^{R}=p_{1j}^{I}=q_{1j}^{I}=\frac{1}{64},$$

$$p_{2j}^{R}=q_{2j}^{R}=p_{2j}^{I}=q_{2j}^{I}=1,\bar{p}_{1j}^{R}=\bar{q}_{1j}^{R}=\bar{p}_{1j}^{I}=\bar{q}_{1j}^{I}=\frac{1}{48},\bar{p}_{2j}^{R}=\bar{q}_{2j}^{R}=\bar{p}_{2j}^{I}=\bar{q}_{2j}^{I}=1,$$

$$\tilde{p}_{1i}^{R}=\tilde{q}_{1i}^{R}=\tilde{p}_{1i}^{I}=\tilde{q}_{1i}^{I}=\frac{1}{32},\tilde{p}_{2i}^{R}=\tilde{q}_{2i}^{R}=\tilde{p}_{2i}^{I}=\tilde{q}_{2i}^{I}=1,\hat{p}_{1i}^{R}=\hat{q}_{1i}^{R}=\hat{p}_{1i}^{I}=\hat{q}_{1i}^{I}=\frac{1}{96},\hat{p}_{2i}^{R}=\hat{q}_{2i}^{R}=\hat{p}_{2i}^{I}=\hat{q}_{2i}^{I}=1,$$

$$\bar{a}_{11}^{R}=0.1,\bar{a}_{11}^{I}=0.2,\bar{a}_{12}^{R}=0.2,\bar{a}_{12}^{I}=0.1,\bar{a}_{21}^{R}=0.3,\bar{a}_{21}^{I}=0.5,\bar{a}_{22}^{R}=0.5,\bar{a}_{22}^{I}=0.3,\bar{\alpha}_{11}^{R}=0.2,$$

$$\bar{\alpha}_{11}^{I}=0.4,\bar{\alpha}_{12}^{R}=0.4,\bar{\alpha}_{12}^{I}=0.2,\bar{\alpha}_{21}^{R}=0.1,\bar{\alpha}_{21}^{I}=0.3,\bar{\alpha}_{22}^{R}=0.3,\bar{\alpha}_{22}^{I}=0.1,\bar{b}_{11}^{R}=0.6,\bar{b}_{11}^{I}=0.3,$$

$$\bar{b}_{12}^{R}=0.3,\bar{b}_{12}^{I}=0.6,\bar{b}_{21}^{R}=0.2,\bar{b}_{21}^{I}=0.5,\bar{b}_{22}^{R}=0.5,\bar{b}_{22}^{I}=0.2,\bar{l}_{11}^{R}=0.2,\bar{l}_{11}^{I}=0.3,\bar{l}_{12}^{R}=0.3,$$

$$\bar{l}_{12}^{I}=0.2,\bar{l}_{21}^{R}=0.1,\bar{l}_{21}^{I}=0.3,\bar{l}_{22}^{R}=0.2,\bar{l}_{22}^{I}=0.5,\bar{I}_{1}^{R}=2,\bar{I}_{1}^{I}=3,\bar{I}_{2}^{R}=3,\bar{I}_{2}^{I}=2,\bar{J}_{1}^{R}=1,$$

$$\bar{J}_{1}^{I}=2,\bar{J}_{2}^{R}=2,\bar{J}_{2}^{I}=1,\beta=\frac{1}{2},\delta_1^{R}=\delta_1^{I}=\frac{3}{40},\delta_2^{R}=\delta_2^{I}=\frac{1}{12},\sigma_1^{R}=\sigma_1^{I}=\frac{2}{15},\sigma_2^{R}=\sigma_2^{I}=\frac{53}{480},$$

$$\gamma=\frac{2}{105},\frac{\beta}{1-\gamma}=\frac{105}{206},\mu_1^{R}=\mu_1^{I}=\frac{533}{3090},\mu_2^{R}=\mu_2^{I}=\frac{52}{309},\vartheta_1^{R}=\vartheta_1^{I}=\frac{416}{1545},\vartheta_2^{R}=\vartheta_2^{I}=\frac{689}{3090},\lambda=\frac{416}{10\,815}。$$

因为 $\lambda<1$，所以由定理 1 可得，系统(15)在 $\Omega^*=\{\boldsymbol{\psi}|\boldsymbol{\psi}\in\Omega,\|\boldsymbol{\psi}\|\leqslant\frac{105}{206}\}$ 中存在唯一的、一致稳定的概周期解。

参考文献：

[1] 周立群，宋协慧. 一类具比例时滞脉冲递归神经网络的全局多项式稳定性[J]. 电子科技大学学报，2021，50(1)：91-100.

[2] 刘新，陈丽丽，黄帅. 一类随机细胞神经网络的均方指数稳定性[J]. 哈尔滨理工大学学报，2020，25(5)：149-157.

[3] 宋协慧，周立群. 一类具多比例时滞脉冲递归神经网络的稳定性分析[J]. 天津师范大学学报(自然科学版)，2020，40(5)：1-8.

[4] 刘锦，赵维锐. 具有时变时滞的中立型神经网络的稳定性分析[J]. 华东师范大学学报(自然科学版)，2020(4)：35-44.

[5] 徐西睿. 改进神经网络的舰船电力系统稳定性容错控制[J]. 船舰科学技术，2020，42(14)：115-117.

[6] NITTA T. Solving the XOR problem and the detection of symmetry using a single complex-valued neuron[J]. Neural networks，2003，16(8)：1101-1105.

[7] 冯靓，胡成，于娟. 脉冲耦合复值神经网络的全局渐进同步[J]. 西南师范大学学报(自然科学版)，2022，47(3)：9-15.

[8] 陈宇,周博,宋乾坤.具有不确定性的分数阶时滞复值神经网络无源性[J].应用数学和力学,2021,42(5):492-499.

[9] 刘苑醒,张玮玮,张红梅.分数阶复值神经网络的准一致同步[J].安庆师范大学学报(自然科学版),2020,26(3):6-15.

[10] 舒含奇,宋乾坤.带有时滞的 Clifford 值神经网络的全局指数稳定性[J].应用数学和力学,2017,38(5):513-525.

[11] WAN P,JIAN J G. Impulsive stabilization and synchronization of fractional-order complex-valued neural networks[J]. Neural processing letters,2019,50:2201-2208.

[12] 方聪娜,宾红华.具有时变时滞的复值神经网络的概周期解[J].厦门大学学报(自然科学版),2021,60(6):989-995.

[13] FINK A M. Almost periodic differential equations[M]. New York:Springer-Verlag,1974:121-126.

[14] 何崇佑.概周期微分方程[M].北京:高等教育出版社,1992:80-94.

[15] WANG Q Y. The existence and uniqueness and stability of almost periodic solutions[J]. Acta mathematics sinica,1997,40(1):80-89.

The Almost Periodic Solution of Complex-valued BAM Neural Networks with Time-varying Delays

ZHAO Lili

(*School of Mathematics and Statistics*,*Yunnan University*,*Kunming* 650091,*China*)

Abstract:In this paper,a class of complex-valued BAM neural networks with time-varying delays is studied. By using the Fixed-point theorem in Banach spaces,the exponential dichotomy on the set of real numbers,and some techniques of differential inequalities,sufficient conditions for the existence,uniqueness and uniform stability of almost periodic solutions of the complex-valued neural networks are obtained. Finally,an example is given to verify the validity and feasibility of the results.

Keywords:complex-valued neural networks;BAM neural networks;almost periodic solution;fixed point theorem;uniform stability

(责任编辑:贾晶晶)